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AN

EASY and COMPENDIOUS INTRODUCTION TO Arithmetic and Algebra.

IN TWO PARTS.

The Firſt contains ARITHMETIC,
VULGAR and DECIMAL;

With Variety of Examples to explain each Rule: And the Method of ſquaring Dimenſions, called Croſs Multiplication; Extraction of the Square and Cube Roots; with their Application to Uſe.

The Second Part ALGEBRA;

Being an eaſy Introduction to that admirable Way of Reaſoning; with the Solution to upwards of One Hundred and Twenty Numerical and Geometrical Queſtions.

THE WHOLE
Delivered in a plain and familiar Manner; being a very uſeful Companion for all Students in ARITHMETIC and ALGEBRA.

By RICHARD CARR.

LONDON:
Printed for the AUTHOR.
MDCCLXI.

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PREFACE.

A Great many have wrote upon this *Subject*, and moſt have met with deſerved Succeſs; but how far this may merit the Approbation of the Public, Time alone muſt determine. I only wiſh it may meet with impartial Judges, who will undoubtedly give it a Character equal to its Deſerts, and then I have Reaſon to believe it will anſwer the End deſigned; that is to be a Guide to thoſe who want to attain a Knowledge of ſuch uſeful aud entertaining Studies, and an agreeable Companion for thoſe who have made ſome Progreſs in them. The fundamental Rules of Arithmetic are laid down ſo plain, that the meaneſt Capacity, with a little Application, may underſtand them; and when I come to the Rule of Three, I make uſe of a few Characters, which the Reader may ſoon be Maſter of; and then he may proceed with Pleaſure. What Rules are omitted, have their Dependance on the Rules of Proportion, which are fully explained; and as moſt Queſtions in Practice are done with much Eaſe and Certainty, by the Multiplication here made uſe of; I hope the Want of them will not be conſidered as a Defect. I have very plainly and intelligibly delivered the Algebraical Part, and exemplified the Rules with a Collection of Queſtions, anſwered in a plain and familiar Manner. I have not notified in the Margin when the Reader is to make Uſe of a Step, *or an abſolute Number; but the Step following will determine which it is.*

Care has been taken to keep this Treatiſe as free as poſſible from the Errors of the Preſs; but if there ſhould be any, I hope the Reader will correct and excuſe them. The Queſtions are moſtly anſwered numerically, being the moſt eaſy Way to familiariſe the Scholar to the Uſe of Algebra, which when underſtood, he may uſe the literal way with pleaſure.

NUMERATION

TEaches to place, or give the juſt Value, to any Number demanded; to do which, obſerve the following Table.

The TABLE.

1	Units.
2 1	Tens.
3 2 1	Hundreds.
4 3 2 1	Thouſands.
5 4 3 2 1	T. of Th.
6 5 4 3 2 1	C. of Th.
7 6 5 4 3 2 1	Millions.
8 7 6 5 4 3 2 1	T. of M.
9 8 7 6 5 4 3 2 1	C. of M.

9 8 7 6 5 4 3 2 1

In the Table above you ſee how each Place exceeds the former ten times, increaſing towards the left Hand. The firſt is the Place of Units, the ſecond of Tens, the third of Hundreds, &c.

ADDITION

Is the gathering ſeveral Numbers into one, which is then called the Sum or the Aggregate; as 6 and 8 are 14.

Addition begins at the right Hand, and adds the particular Sums of the ſeveral Rows underneath every one in its own Place.

EXAMPLES.

Ex. 1ſt.	Ex. 1d.	Ex. 3d.
2743	2435	2374
5427	1437	5615
6514	6214	8731
3215	3715	6425
6732	7214	7431
5475	3746	5762
30106	24761	36338

In the firſt *Example*, the Sum of the firſt Row is 26; I place down the 6 under itſelf, and carry 2, being the Number of the Tens to the next Row, and find the Sum of it to be twenty, then I place down a Cypher, and carry two to the next Row, and find the Sum to be 31. I place down the 1, and carry 3 to the laſt Row, and the Sum is 30, which I place down, and it is done. And ſo of the reſt.

ADDITION of MONEY.

N. B. Four Farthings is one Penny, Twelve Pence one Shilling, and Twenty Shillings one Pound.

EXAMPLES.

l.	s.	d.	l.	s.	d.
27	14	11½	374	17	6½
15	16	4¼	542	15	11¼
72	17	3¾	364	16	7¾
56	18	10½	273	18	3½
32	11	9¼	614	15	7¼
64	16	7¾	364	19	3½
37	14	6½	764	13	5¼
67	19	2¼	527	11	10½
376	09	7¾			

In the firſt *Example.* The Farthings are 15, which is three Pence three Farthings, put down ¾ under the Farthings, and carry three to the Pence place, and the Sum of the Pence is 55, which is 4 Shillings and 7 Pence; place the 7 under the Pence, and carry 4 to the Shillings, and the Sum is 129 Shillings, which is 6 Pounds 9 Shillings; place down the 9, and carry 6 to the Pounds, then proceed as in the laſt Examples of whole Numbers.

l.	s.	d:	l.	s.	d.
375	17	4 ½	742	14	3 ½
654	14	9 ¼	567	15	7 ¾
276	11	4 ½	254	18	6 ¼
346	14	7 ¾	675	17	8 ½
214	16	2 ½	792	15	7 ¾

Addition is proved by making a ſecond Addition of the ſame Sums, omitting the top Line; then adding the Sum to the top Line, if it makes the Sum of the whole it is right.

N. B. Addition of Time, Weight, Meaſure, &c. &c. is performed in the ſame Manner, only obſerving how many of the inferior makes one of the next, aſcending by the Tables after Diviſion.

* * *

SUBTRACTION

IS the taking a ſmall Number from a large one, and the Number found is called the Remains, the Exceſs or the Difference. As from 12 take 4, and there remains 8.

EXAMPLES.

Example 1ſt.		Example 2d.	
From	37 427	From	37 427
Take	21 314	Take	14 678
Remains	16 113	Remains	22 749
Proof	37 427	Proof	37 427

N. B. In the firſt *Example*, I take 4 from 7, and there remains 3, and ſo place it down, and proceed to the End of that Example.

But in the ſecond *Example*, I cannot take 8 from 7, ſo I borrow 10 and put it to the 7, and it makes 17, then 8 from 17, and there reſts 9, which I place down, and carry one that I borrowed to the 7, and it makes 8; then 8 from 2 I cannot, but I borrow 10, and add it to the 2, and it makes 12; then 8 from 12, and there remains 4, which place down; and ſo proceed to the End, remembring to carry one to the next Figure when you borrow the 10.

If you add the Remains to your leaſt given Number, the Sum will be the greater Number given, which is the Proof.

	l.	*s.*	*d.*		*l.*	*s.*	*d.*
From	37	17	11 ½	From	274	19	03 ¾
Take	24	13	10 ¼	Take	175	16	04 ½
Differ.	13	4	1 ¼	Remains	99	02	11 ¼
Proof	37	17	11 ½	Proof	274	19	03 ¾

N. B. What you carry at in *Addition*, you borrow at in *Subtraction*.

A Gentleman bought an Eſtate, valued at 7427 *l.* 17 *s.* 11 *d.* and his Caſh is only 1498 *l.* 18 *s.* 11 *d.* ¾, how much muſt he ſend for to his Banker to be enabled to pay for it? Place the Sums thus:

	l.	*s.*	*d.*
Value of the Eſtate - -	7427	17	11
The Caſh at home - -	1498	18	11 ¾
This comes from the Banker	5928	18	11 ¼
Proof	7427	17	11

N. B. The Caſh brought from the Banker, being added to the Caſh at home, muſt be the Value of the Eſtate.

What Sum of Money added to 374 *l.* 11 *s.* 5 *d.* ½, will make it 1000 *l.* Place it as follows:

l.	*s.*	*d.*	
1000	00	00	
374	11	05 ½	
625	08	06 ½	this is the Sum you muſt add.
1000	00	00	Proof.

What is the Difference betwixt 207*l.* 14*s.* 2*d.* and 176*l.* 19*s.* 11*d.*

	l.	*s.*	*d.*
	207	14	02
Leaſt Sum	176	19	11 ½
Difference	30	14	02 ½
Proof	207	14	02

A Steward received the following Sums for his Lord; from *A.* 172 *l.* 17 *s.* 11 *d.* ½; from *B.* 160 *l.* 14 *s.* 8 *d.* from *C.* 270 *l.* 11 *s.* 4 *d.* from *D.* 120*l.* 11 *s.* 6 *d.* from *E.* 192 *l.* 11 *s.* 8 *d.* ½. And he ſent to his Lord Bank Notes, Value 407 *l.* 17 *s.* 11 *d.* Paid Workmen 62 *l.* 17 *s.* 11 *d.* Paid the Wine-Merchant 42 *l.* 17 *s.* 00 *d.* His own Salary for a Quarter 12 *l.* 10 *s.* 00 *d.* What remains of his Lord's in his Hands?

From *A.*	172 : 17 : 11½
From *B.*	160 : 14 : 8
From *C.*	270 : 11 : 4
From *D.*	120 : 11 : 6
From *E.*	192 : 11 : 8½
Tot. rec.	917 : 07 : 02
Diſburſt.	526 : 02 : 10
Ballance.	391 : 04 : 04

Sent in Notes	407 : 17 : 11
P. Workmen	62 : 17 : 11
W. Merchant	42 : 17 : 00
Salary	12 : 10 : 00
T. laid out.	526 : 02 : 10
Money in Hand.	391 : 04 : 04
Proof	917 : 07 : 02

If the Tower of *London* was built by the *Conqueror* in the Year 1077, how many Years is it ſince, this Year being 1750.

The

The preſent Year - -	1750
The Year when the Tower was built	1077
Years ſince the Tower was built	673
Proof	1750

If the Time any Building has ſtood be added to the Year it was built in, it makes the preſent Year.

An Apprentice was bound for 7 Years, but he runs away at the end of 6 Years, 6 Months, 6 Weeks, 6 Days, 6 Hours, 6 Minutes, the Queſtion is, how long he had to ſerve ?

	Y.	M.	W.	D.	H.	M.
The Time he was to ſerve is,	7	0	0	0	0	0
The Time he did ſerve was,	6	7	2	6	6	6
The Time he had to ſerve	0	5	1	0	17	54
Proof	7	0	0	0	0	0

N. B. The 6 Weeks is 1 Month and 2 Weeks, and 13 Months is one Year.

MULTIPLICATIO

IS that by which we find the Increaſe or Amount of any Number, being ſo many Times taken as there are Units in another Number, and this Increaſe or Amount is called the Product. The Factus, the Rectangle, or the Plain ; and the Numbers producing

The TABLE.

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

Find any Number in the Top of the Table, and the Number you would multiply it by, in the Side, and in the Square where they meet is the Product. Suppoſe you would multiply 7 by 8, find the 8 at the Top, and the 7 in the Side, and in the Square where they meet ſtands 56 the Product, and ſo of any other.

Mind to have the Table off by Heart, before you go any farther.

Multiply 674274 { This is the Multiplicand.
By 6 } This the Multiplier.

4045644 This is the Product, &c.

In the above Example I ſay, 6 times 4 is 24 ; put down 4 and carry 2 to the next Place, ſaying 6 times 7 is 42, and 2 I carry is 44 ; put down 4 and carry 4. And proceed in this Manner to the End, putting down the odd Numbers, and carry one for every Ten that you have to the next Place.

Multiply 576274
By 37

4033918
1728822

21322138

N. B. You muſt place the Beginning of each Multiplication under its own Factor, as you may obſerve in the above Example ; when I began to multiply by 3, I put the firſt Figure of its Product, being 2, under itſelf, and then proceeded as before, and the two Multiplications added into one Sum, is the Product.

Multiply 674076
By 346

4044456
2696304
2022228

233230296

Multiply 47247
By 3004

188988
14174100

141929988

In the above Example, the Cyphers only fill up the Places under which they ſtand, and then you begin your Multiplication by the 3.

Multiply 674264
By 300

202279200

Multiply 721462
By 7400

2885848OO
5050234

5338818800

N. B. If your Multiplier have Cyphers to the right Hand, it is only multiplying by the Figures, and adding the Cyphers to the Product.

Multiply 67424
By 1000

67424000

To multiply by an Unit with any Number of Cyphers; it is only adding the Cyphers to the Multiplicand.

Multiply	74624	Multiply	5742
By	15		17
	1119360		97614

This Multiplication is performed by taking in the back Figure, thus, as in the firſt Example, I ſay 5 times 4 is 20, put down 0 and carry 2, then 5 times 2 is 10, and 2 I carry is 12, and the back Figure 4, is 16, put down 6 and carry 1; then 5 times 6 is 30, and 1 I carry is 31, and 2 the back Figure is 33, put down 3 and carry 3; then 5 times 4 is 20, and 3 is 23, and the back Figure 6 is 29, put down 9 carry 2; then 5 times 7 is 35 and 2 is 37, and the back Figure 4 is 41, put down 1 and carry 4. Now as the Multiplication by 5 is ended, I add the 4 I carry to the 7 and it makes 11, which I put down, and the Work is ended; and ſo of any other.

Multiply	54762	Multiply	75642
By	19	By	18
	1040478		1361556

Multiplication by two back Figures is performed by the former Rule.

EXAMPLES.

EXAMPLES.

Multiply 742657	Multiply 67542
By 1714	1619
10397198	1283298
12625169	1080672
1272914098	109350498

Multiplication may be performed by Factors, thus. Take any two or three Numbers, &c. which when multiplied into one another, will produce the Multiplier given; and multiply the Multiplicand by one of them, and that Product by the other, &c. and the laſt Multiplication will be the deſired Product.

EXAMPLES.

Multiply 74276 by 24: The Factors are 4 and 6.
6
445656
4
1782624

Multiply 67424 by 48. The Factors are 8 and 6.
8
539392
6
3236352

N. B. The laſt Product had been the ſame, if I had taken any other Numbers; which when multiplied together, would have made the Multiplier; as 3, and 8, in the firſt Example. And 4, 6, and 2 in the ſecond.

Multiplication of Compound Quantities.

If one Pound of Tea coſt 5 *s.* 9 *d.* what will eight Pounds coſt?

5 : 9
8

l. 2 : 6 : 0

I ſay 8 times 9 is 72, which is 72 Pence, that is 6 Shillings, put a Cypher under the Pence place, and carry 6, ſaying 8 times 5 is 40, and 6 I carry is 46, which is 46 Shillings, equal to 2 *l.* 6 *s.* 0 *d.* the Anſwer.

If one Yard of Velvet coſt 1 *l.* 4 *s.* 6½ *d.* What will 9 Yards coſt at that rate?

1 : 4 : 6½
9

11 : 0 : 10½ The Anſwer.

For 9 Halfpence is 4 *d.* ½; I ſet down the ½, and carry 4 to the Pence place, ſaying 9 times 6 is 54, and 4 I carry is 58 Pence, which is 4 *s.* 10 *d.* put down the 10 and carry 4, ſaying 9 times 4 is 36, and 4 I carry is 40 Shillings, put down a Cypher in the Shillings place, and carry 2 to the Pounds place, ſaying 9 times 1 is 9, and 2 I carry is 11 Pounds, which I put down, and the Anſwer is 11*l.* 0*s.* 10*d.* ½. And ſo of all the reſt of the Examples.

If 1 Yard coſt 3*s.* 7*d.* ¼, what will 24 Yards coſt at that rate?

3 : 7¼
3
10 : 9¾
8
4 : 6 : 6

N. B. The Factors are 3 and 8, or 4 and 6; ſo that multiplying firſt by 3, and that Product by 8, gives the Anſwer.

If 1 Pound of Tea coſt 7*s.* 4*d.* ½, what will 56 Pounds coſt at that rate?

7 : 4½
8
2 : 19 : 0
7
20 : 13 : 0

In anſwering Queſtions of this Kind, take any two Numbers, which when multiplied together will produce the Number given, and multiply the Price by one of the Factors, and that Product by the other, and the laſt Product will be the Anſwer, as in the two laſt Examples. In the firſt I multiply the Price of 1 Yard by 3, which gives me the Price of 3 Yards, then I multiply the Price of 3 Yards by 8, and it gives me the Price of 8 times 3 Yards, which is 24 Yards. If I had multiplied the Price of 1 Yard by 4, and that Product by 6, it would have been the ſame Anſwer. And in the ſecond Example, I multiply firſt by 7, and then by 8 for the ſame Reaſon.

If 1 Yard coſt 2 *s.* 3 *d.* what will 60 Yards coſt at that Price?

2 : 3
10
1 : 2 : 6 This is the Price of 10 Yards.
6
6 : 15 : 0 The Price of 60 Yards.

If a parcel of Cloth coſt 8 *l.* 16 *s.* 5 *d.* what will 336 Parcels coſt at that rate?

8 : 16 : 5	
10	
88 : 4 : 2	The Price of 10 Parcels.
10	
882 : 01 : 8	The Price of 100
3	
2646 : 05 : 0	The Price of 300
264 : 12 : 6	The Price of 30
52 : 18 : 6	The Price of 6
2963 : 16 : 0	The Price of 336

Firſt, I multiply the Price of 1 Parcel by 10, which gives me the Price of 10 Parcels, then I multiply that Product by 10, and it gives the Price of 10 times 10 Parcels, or 100; which Product multiplied by 3 gives the Price of 300 Parcels. Then multiplying the Price of 10 Parcels by 3, it gives the Price of 30 Parcels, which place under the Price of 300 Parcels. Laſtly, multiply the Price of one Parcel by 6, and place it under the laſt, then thoſe 3 Sums added together gives the Anſwer equal, 2963 *l.* 16 *s.*

If 1 Gallon of Rum coſt 8 *s.* 6 *d.* what will 6742 coſt?

s. d.	
8 : 6	The Price of 1 Gallon.
10	
l. 4 : 5 : 0	The Price of 10 Gallons.
10	
42 : 10 : 0	The Price of 100 Gallons.
10	
425 : 00 : 0	The Price of 1000 Gallons.
6	
2550 : 00 : 0	The Price of 6000 Gallons.
297 : 10 : 0	The Price of 700 Gallons.
17 : 00 : 0	The Price of 40 Gallons.
00 : 17 : 0	The Price of 2 Gallons.
2865 : 07 : 0	The Price of 6742 Gallons.

This Method of finding the Value of any Number of Yards, Gallons, Ells, Pounds, &c. at any Price *per* Yard, Gallon, &c. is of great Uſe for finding the Value in a ſhort and eaſy manner, and very often exceeds the Method of Practice; and will be of excellent Uſe for ſuch as buy or ſell by Retail, as no Multiplication can be eaſier than multiplying by 10.

If a Hogſhead of Tobacco coſt 3 l. 8 s. 4 d. ½, what is the Value of 432 Hogſheads?

l.	s.	d.	
3	8	4½	
		10	
34	03	09	The Price of 10.
		10	
341	17	06	The Price of 100.
		4	
1367	10	00	The Price of 400
102	11	03	The Price of 30
6	16	09	The Price of 2
1476	18	00	The Price of 432

Croſs Multiplication; or, the Method of ſquaring Dimenſions.

This differs very little from Compound Multiplication, only in placing the Figures of your Multiplicand and Multiplier. Examples will make it very eaſy. Remember to carry 1 at every 12 to the next ſuperior Name, excepting when you multiply Feet by Feet, and then you carry 1 at every 10, as in common Multiplication.

EXAMPLES.

EXAMPLES.

If a Board be 1 Foot 4 Inches broad, and 13 Feet long, how many Feet is contained therein? Place it thus.

F. I.
13 : 0 Length.
1 : 4 Breadth.

4 : 4 : 0
13 : 0

17 : 4 : 0 The Anſwer.

If a Board be 8 Feet 6 Inches long, and 3 Feet 4 Inches broad, what is the ſuperficial Content?

F. I. P.
8 : 6
3 : 4

2 : 10 : 0
25 : 6 : 0

28 : 4 : 0 The Anſwer.

F. I. P. S. T. F. I. P. S. T.
Multiply 8 : 4 : 7 : 3 : 5 by 4 : 2 : 6 : 4 : 3
Place it thus 4 : 2 : 6 : 4 : 3

2 : 1 : 1 : 9 : 10 : 3
2 : 9 : 6 : 5 : 1 : 8
4 : 2 : 3 : 7 : 8 : 6
1 : 4 : 9 : 2 : 6 : 10
33 : 6 : 5 : 1 : 8

35 : 3 : 7 : 7 : 6 : 1 : 5 : 6 : 3

Multiply 259 Feet 10 Inches and 8 Parts, by 18 Feet 5 Inches and 4 Parts.

	F.	I.	P.	S.	T.
	259	10	08		
			18	5	4
	7	2	7	6	8
	108	3	5	4	
Firſt multiplied by Ten	2598	10	8		
Firſt multiplied by Eight	2079	1	4		
The Product	4793	6	0	10	8

If the Learner mind to carry 1 for every 12 to the next ſuperior Place, ſetting down the Remains under 12 in its own Place, (excepting when he multiplies Feet by Feet, which he muſt carry 1 for every 10, as in common Multiplication) he cannot go wrong; for the placing the Multiplicand and Multiplier in this Manner, ſhews at once the Name of every particular Product, as in the above Example, Parts multiplied by Parts, give Thirds in the Product; that is, 4 times 8 is 32 Thirds, which is 2 Seconds and 8 Thirds; put down the 8 and carry 2, ſaying, 4 times 10 is 40, and 2 is 42, which is 6 Seconds and 3 Parts, put down the 6 and carry 3; then multiply 259 Feet by 4 Parts, and the 3 Parts that you carry makes 1039 Parts, which divided by 12 gives 86 Inches and 7 Parts remaining; then the Inches divided by 12, gives 7 Feet and 2 Inches remaining, which place down in their own Places, and it is done. So proceed with the reſt.

If a Board be 2 Feet 6 Inches ſquare, what is the ſuperficial Content?

F. I.
2 : 6
2 : 6

1 : 3 : 0
5 : 0

6 : 3 : 0 The Anſwer.

If a Plank be 8 Feet 7 Inches and 6 Parts long, and 4 Feet 6 Inches and 4 Parts broad, how many ſquare Feet is contained in that Plank?

F. I. P. S. T.
8 : 7 : 6
4 : 6 : 4

0 : 2 : 10 : 6 : 0
4 : 3 : 9 : 0
34 : 6 : 0

The Anſwer 39 : 0 : 7 : 6 : 0

If a Room be 37 Feet 4 Inches and 8 Parts long, and 28 Feet 7 Inches and 6 Parts broad, how many ſquare Feet is contained in that Room?

F. I. P.		F. I. P.
28 : 7 : 6	Breadth.	
37 : 4 : 8	Length.	
1 : 7 : 1 : 0 : 0	Top mult. by	00 0 8
9 : 6 : 6 : 0	Top mult. by	00 4 0
200 : 4 : 6	Top mult. by	07 0 0
286 : 3 : 0	Top mult. by	10 0 0
572 : 6 : 0	The laſt mult: by 2 equals	20 0 0
1070 : 3 : 1 : 0 : 0	Content	37 4 8

N. B. The laſt multiplied by 2 is the ſame as if the Top was multiplied by 20, becauſe twice 10 is 20. And the Addition to the right Hand, ſhews that the ſeveral Multiplicators added, is the ſame as your firſt Multiplicator or Length. If the Learner poſtpone this till he is perfect in Diviſion, it will be a great deal more plain and eaſy.

Let there be a Piece of Timber in Form of a Parallelopipedon, whoſe length is 8 Foot 6 Inches, and the ſide of the Square its Baſe 2 Foot 6 Inches; I demand its Solidity.

2 : 6 Side of the Baſe.
2 : 6

1 : 3 : 0
5 : 0

6 : 3 : 0
8 : 6

3 : 1 : 6
50 : 0 :

53 : 1 : 6 Solidity.

DIVISION

IS a Rule by which we diſcover how many times one Number (called the Diviſor) is contained in another (called the Dividend) and the Number found is called the Quotient. And ſometimes there is a fourth Number called the Remainder.

N. B. The Quotient ſo often contains an Unit, as the Dividend contains the Diviſor.

EXAMPLES.

Divide 24 Pounds amongſt 4 Men.

Place them in this Manner.

```
4)24(6
  24
  ———
```

N. B. 24 is the Dividend, 4 is the Diviſor, and 6 is the Quotient, found by dividing the Dividend by the Diviſor ; ſaying how many times 4 in 24, which is 6 times, and then multiplying the Quotient Figure by the Diviſor, and place the Product under the Dividend which is 24, and nothing remains, and the Quotient is 6 Pounds *per* Man.

If 6678 Pounds be equally divided amongſt 9 Men, what is each Man's Share?

```
9)6678(742
  63
  ——
   37
   36
   ——
    18
    18
    ——
     0
```

I ſay how many times 9 in 66, and I find 7 times 9 is 63, which I place under the 66, and ſubtract, and find the Difference 3, to which I bring down the 7, and make it 37; then I find how many times 9 in 37, and it is 4 times 9, which is 36, which I place under 37, and ſubtract, and find the Difference 1, to which I bring down the 8, and make it 18; then how many times 9 in 18, which is 2 times, which I put in the Quotient, and multiply the Diviſor by it, and find it 18, which I place under 18, and find no Remainder; and each Man's Share is 742 Pounds.

Diviſion is the only true way to prove Multiplication; for if one divide the Product by the Multiplier, the Quotient will be the Multiplicand, and dividing the Product by the Multiplicand, the Quotient will be the Multiplier.

So is Multiplication the Way to prove Diviſion; for multiplying the Quotient by the Diviſor, the Product produced will be the Dividend. *N. B.* If there be a Remainder it muſt be added to the Product, and the Sum will be the Dividend.

This leads to a ſhort and eaſy Method to be Maſter of Diviſion; as for Example, I multiply 742 by 24. Place it thus:

742	24)17808(742
24	168
2968	100
1484	96
17808	48
	48
	00

I firſt multiply and find the Product 17808, which I divide by 24, and I know the Quotient will be 742; yet I proceed in the Form of Diviſion, multiplying every Quotient Figure into the Diviſor; by this Means you will make the Form of Diviſion familiar.

Multiply 5746
By 28

45968
11492

160888

28)160888(5746
140

208
196

128
112

168
168

0

Multiply 67432
By 507

472024
3371600

34188024

507)34188024(67432
3042

3768
3549

2190
2028

1622
1521

1014
1014

0

Thus by taking promiſcuouſly any two Numbers, and multiplying them together, and then dividing the Product by one of them, you will have the other in the Quotient; and by doing in this Manner,

ner, you will facilitate Diviſion, and ſooner become Maſter of it, than by any other Direction.

Multiply 60407
By 2004

241628
12081400

121055628

Divide 121055628 by 60407

60407)121055628(2004
120814

241628
241628

0

N. B. That every Figure you bring down from the Dividend to the Remainder, if you cannot have the Diviſor in it, you muſt place a Cypher in the Quotient, as in the above Example: After I can have my Diviſor twice in the firſt Part of my Dividend, and there remains 241; then I bring down the 6 from the Dividend to it, and it is 2416; but I cannot have my Diviſor in it, ſo I place a Cypher in the Quotient, and bring down the next Figure 2 to it, and ſtill find I cannot have my Diviſor in it; ſo I place a Cypher in the Quotient, and bring down my laſt Figure 8, and find it contains my Diviſor 4 times, and there is no Remainder.

A Gentleman left 740 Pounds to be divided amongſt 10 Servants. What is each Man's Share?

1|0)74|0(74 Pounds each Man.
74

To divide any Number by 10, is only cutting from the Dividend a Figure to the right Hand, and the Remainder of the Dividend is the Quotient. If your Diviſor is 100, cut 2 from the Dividend; if 1000, cut 3 Figures from it, and the Remainder is the Quotient.

EXAMPLES.

Divide 612 Pounds among 100 Men.

1|00)6|12(6 Pounds each.
6

The 12 which is cut from the Dividend is the Remains.

Divide 3742876 by 67400

674|00)37428|76(55
3370
3728
3370
35876 Remains.

67400 Diviſor.
55 Quotient.
337000
337000
3707000 Product.
35876 Remains.
3742876 Proof.

Divide 37100 by 55.

55)37100(674
330
410
385
250
220
30 Remains.

674 Quotient.
55 Diviſor.
3370
3370
37070 Product.
30 Remains.
37100 Proof.

If there be never ſo many Cyphers in the Dividend, and none to the right Hand of your Diviſor, you muſt cut none of; as in this Example.

There are four Cannon, as *A. B. C.* and *D.* appointed at a Battery. *A* ſpends at a Shot 9 Pounds of Powder, *B* 5 Pounds, *C* 4 Pounds, and *D* 2 Pounds; and there is 1400 Pounds of Powder to be ſpent by theſe four Pieces, how many Shots can they make each, and how much Powder did each conſume?

A. ſpends 9 at a Shot.
B. ſpends 5 at a Shot.
C. ſpends 4 at a Shot.
D. ſpends 2 at a Shot.

The Diviſor 20

2|0)140|0(70 Shots each.
14
0

When they were all once diſcharged, they had conſumed 20 Pounds of Powder, then it is plain dividing the Quantity by what they conſume at one Shot, gives in the Quotient how many Shots it will ſupply them with; then multiplying the Number of Shots by what each ſpends at a Shot, gives the Quantity of what each ſpends in the whole.

70 multiplied by 9 gives 630, what *A.* ſpends.
70 multiplied by 5 gives 350, what *B.* ſpends.
70 multiplied by 4 gives 280, what *C.* ſpends.
70 multiplied by 2 gives 140, what *D.* ſpends.

1400 Proof.

Diviſion of Factors is performed in the following Manner.

Divide 43904 by 56

```
7|43904
 ------
 8|6272
  ------
   784
```

Divide 78894 by 81

```
9|78894
 ------
 9|8766
   974
```

In the firſt of the Examples, I divide firſt by 7, and place the Quotient under the Dividend; then I divide that Quotient by 8, and find the next Quotient 784, the ſame as if I had divided the firſt Dividend by 56, becauſe 7 times 8 is 56.

In the ſecond Example, I divide firſt by 9, and that Quotient by 9 again, and find the laſt Quotient 974; becauſe 9 times 9 is 81.

Diviſion

Diviſion of Compound Quantities

May be eaſily performed by the Help of the Multiplication of Tens, in the following Manner. Let the Queſtion to be reſolved be this:

If one Yard coſt 7 s. 4 d. how many Yards may I have for 273 l. 10s. 8 d.?

s. d.	l. s. d.	l. s. d.	
7 : 4 Diviſ. Un.	36 : 13 : 4	273 : 10 : 8	746
10		256 : 13 : 4	
3 : 13 : 4 Diviſ. Ts.	3 : 13 : 4	16 : 17 : 4	
10		14 : 13 : 4	
36 : 13 : 4 Diviſ. H.	7 : 4	2 : 4 : 0	
		2 : 4 : 0	

You ſee by the precedent Example, that the Diviſors change Place every Operation, and gradually deſcend in a ſubdecuple Ratio, but the Quotient continues in the ſame Place; and as the Multiplication of Tens is very eaſy, thoſe Diviſors are eaſily obtained, and may be found to what Place you pleaſe.

If a Piece of Cloth coſt 10 l. 16 s. 8 d. how many Yards are in the ſame, when the Yard is worth 8 s. 4 d. ?

```
  s.  d.                 l. s. d.    l.   s.  d.
  8 : 4  Diviſ. Un.   | 4 : 3 : 4 | 10 : 16 : 8 | 26Yards.
      10              |           |  8 :  6 : 8 |
 ─────────            |           | ─────────── |
4 : 3 : 4 Diviſ. Ten. |     8 : 4 |  2 : 10 : 0 |
                      |           |  2 : 10 : 0 |
                      |           | ─────────── |
                      |           |      0      |
```

How many Yards can I have for 12 l. 11 s. 5 d. ½, at 7 s. 4 d. ¾ *per* Yard ?

```
                            l.   s.   d.     l.   s.    d.
                         | 3 : 13 : 11½ | 12 : 11 : 05½ | 34Ys
                         |              | 11    1 : 10½ |
  s.   d.                |              | ───────────── |
  7 :  4¾ Un. Div.       |     7 :  4¾  |  1 :  9 :  7  |
       10                |              |  1 :  9 :  7  |
 ─────────               |              | ───────────── |
3 : 13 : 11½ T. Diviſ.   |              |  0 :  0 :  0  |
```

The Diviſion of a Pound Troy.

24	Grains.	make	one Penny Wt.
20	Penny Weight.	make	one Ounce.
12	Ounces.	make	one Pound.

14 Ounces, 12 Penny Weight, make 1 Pound Averdupoiſe.

Diviſion of Averdupoiſe Weight.

16	Drams	make	1 Ounce.
16	Ounces		1 Pound.
14	Pound		1 Stone.
28	Pound		¼ Hundred.
56	Pound		½ Hundred.
112	Pound		1 Hundred.
5	Hundred		1 Hogſhead.
10	Hundred		1 Pipe.
20	Hundred		1 Tun, or Load.

Apothecaries Weight.

24	Grains of Wheat	make	1 Scruple.
3	Scruples		1 Dram.
8	Drams		1 Ounce.
12	Ounces		1 Pound.

Long Meaſure.

3	Barley Corns	make	1 Inch.
4	Inches		1 Palm.
12	Inches		1 Foot.
3	Feet		1 Yard.
5	Feet		1 Geomet. Pace.
6	Feet		1 Fathom.
5½	Yards		1 Perch.
40	Perch		1 Furlong.
8	Furlongs		1 Mile.
3	Miles		1 League.

Cloath

Cloth Meaſure.

4	Nails	make	1 Quarter.
4	Quarters		1 Yard.
5	Quarters		1 Ell Engliſh.
3	Quarters		1 Ell Flemiſh.

Dry Meaſure.

2	Pints, or Pounds	make	1 Quart.
2	Quarts		1 Pottle.
2	Pottles		1 Gallon.
2	Gallons		1 Peck.
4	Pecks		1 Buſhel Land Meaſure.
5	Pecks		1 Buſhel Water Meaſure.
4	Buſhels		1 Coomb.
2	Coombs		1 Quarter.
4	Quarters		1 Chalder.
5	Quarters		1 Tun, or Wey.

Time.

60	Seconds	make	1 Minute.
60	Minutes		1 Hour.
24	Hours		1 Day natural.
7	Days		1 Week.
4	Weeks		1 Month.
13	Months 1 Day		1 Year, or 365 Days.

Liquid Meaſure.

2 Pints	make	1 Quart.
2 Quarts		1 Pottle.
2 Pottles		1 Gallon.
8 Gallons		1 Fir. Ale Soap or Her.
9 Gallons		1 Firkin of Beer.
2 Firkins		1 Kilderkin.
2 Kilderkins		1 Barrel.
42 Galllons		1 Tierce.
63 Gallons		1 Hogſhead.
2 Hogſheads		1 Pipe, or Butt.
2 Buts, or 252 Gal.		1 Tun.

REDUCTION

TEaches to bring a Number from one Denomination to another, whereby we know how many of one Denomination are equal to ſo many of another.

Reduction is performed by Multiplication, ſecondly by Diviſion, thirdly, by Multiplication and Diviſion.

EXAMPLES.

Queſtion 1ſt. Reduce 24 l. 7 s. 4 d. $\frac{1}{4}$ into Farthings.

24 : 7 : 4$\frac{1}{4}$
20

487 Shillings.
12

978
487

5848 Pence.
4

23393 Farthings.

I multiply firſt by 20, and as I multiplied I brought in the 7 Shillings, ſo that in 24 l. 7 s. I find 487 Shillings; this I multiplied by 12, the Pence in one Shilling, bringing in the 4 Pence, and I find the Pence 5848; this Number I multiply by 4, the Farthings in one Penny, bringing in the Farthing, and it makes 23393 Farthings.

Queſtion 2d. In 742 Yards, how many Quarters and Nails?

742 Yards.
4

2968 Quarters in 742 Yards.
4

11872 Nails in 742 Yards.

Queſtion 3d. In 21 *C.* 2 *q.* 14 *lb.* how many Ounces?

```
   21 : 2 : 14
    4 Quarters in a C.
   --
   86
   28 Pounds in a Quarter.
  ---
  692
 173
 ----
 2422
   16 Ounces in a Pound.
 ----
14532
2422
-----
38752 Ounces the Anſwer.
```

Queſtion 4th. In 74 Pounds how many Groats?

```
  74
  20 Shillings in a Pound.
----
1480
   3 Groats in a Shilling.
----
4440 Anſwer.
```

Queſtion 5th. In 4 Baggs of Pepper, weighing each 2 *C.* 3 *q.* 14 *lb.* how many Pounds ?

2 : 3 : 14
4
11 Quarters in one Bagg.
28
92
23
322 Pounds in one Bagg.
4
1288 Pounds in four Baggs.

Queſtion 6th. If 5 Eggs be ſold for a Penny, how many will buy a Horſe worth 12 Pounds ?

12 Pounds Value of the Horſe.
20
240
12
2880
5 Eggs for a Penny.
14400 No. of Eggs the Anſwer.

Queſtion 7th. In 74 Hogſheads how many Gallons and Pints ?

74
63 Gallons in a Hogſhead.

222
444

4662 Gallons.
8 Pints in a Gallon.

37296 Pints the *Anſwer.*

Queſtion 8th. How many Yards of *Mancheſter* Tape will pay for an Eſtate valued at 746 l. 17 s. 7 d. when the Tape is ſold for 3 Yards a Penny.

l. s. d.
746 : 17 : 7
20

14937
12

29881
14937

179251
3 Yards of Tape for a Penny.

537753 Yards of Tape the *Anſwer.*

Queſtion 9th. How many Barley Corns will reach from *York* to *London*, it being 198 Miles?

198	
8	
1584	Furlongs.
40	
63360	Poles.
11	
63360	
63360	
696960	Half Yards.
18	
5575680	
696960	
12545280	Inches.
3	
37635840	B. Corns in 198 Miles.

N. B. All the former Queſtions is proved by dividing each Product by what you multiplied by, and it is called *Reduction* by *Diviſion*.

Queſtion 10th. In 73919 Farthings, how many Pence, Shillings, and Pounds?

```
                 (12)        2|0
    4 | 73919 | 18479 | 153|9 | 76 Pounds.
         4        12      14
        ---      ----    ----
         33       64      13
         32       60      12
        ----     ----    ----
          19       47      19   Shillings.
          16       36
         ----     ----
          31      119
          28      108
          ---     ----
           39      11   Pence.
           36
           ---
            3   Farthings.
```

N. B. I divide firſt by 4, the Farthings in one Penny; the Quotient is the Number of Pence, and the Remains is Farthings; then that Quotient I divide by 12, the Pence in one Shilling, and place it above the Pence, and the Quotient is Shillings, and the Remains Pence; then dividing that Quotient by 20, it gives the Pounds, and the Remains is Shillings; and the Work is done.

Queſtion 11th. A Gentleman left 5 Pounds to be equally divided amongſt Eight Perſons, what is the Share of each ?

```
    l.
    5
    20
   ----
 8)100(12
    8
  -----
   20
   16
  -----
     4
    12
  -----
  8)48(6
    48
   ----
    0
```

N. B. If the Diviſor be larger than the Dividend, the Dividend muſt be brought into the next inferior Denomination, as in this Example ; 5 Pounds is brought into Shillings, and that Product is divided by 8, and the Quotient is 12, and 4 remains, which is brought into Pence, and the Pence divided by 8, and the Quotient is 6, ſo that each Perſon had 12 s. 6 d. the Anſwer.

Obſerve the ſame in all Queſtions of the ſame Kind.

Queſtion 12th. In 37 l. 15 s. how many Sixpences, Fourpences, and Twopences, and of all an equal Number ?

d.	l.	s.
6	37 :	15
4	20	
2	——	
—	755	
12 Diviſor.	12	

——
1510
755
——
12)9060(755.
84
——
66
60
——
60
60
——
0

The Reaſon I divide by 12, is becauſe I find one Six-pence, one Four-pence, and one Two-pence is Twelve-pence; ſo that this Queſtion is only to find how many Shillings is in 37 l. 15 s. ; for if one have a Shilling, he has as much as one that has a Six-pence, one Four-pence, and one Two pence.

Explanation

Explanation of the Symbols made Uſe of in this Arithmetick.

$+$	Signifies Addition, as $7 + 4 + 8 = 19$.
$-$	Signifies Subſtraction, as $18 - 7 = 11$.
$\times$	Signifies Multiplication, as $9 \times 7 = 63$.
$\div$	Signifies Diviſion, as $48 \div 6 = 8$.
$=$	Signifies Equality, as $7 + 8 + 4 = 19$.
$:$	As, or give.
$::$	Signifies ſo is, or ſo will.

If $4 : 6 :: 8 : 12$, that is, if 4 give 6, ſo will 8 give 12.

N. B. If the Sign of Addition be ſet after or betwixt any Number of Figures, or Quantities, it ſhews they muſt be added into one Sum thus; $7 + 8 + 12 + 14$, when added their Sum will be 41. And ſo of any other.

And if the Sign of Multiplication be placed in the ſame Order, they muſt be multiplied into one another, as in this, $8 \times 7 \times 4 \times 6$, the Product will be 1344.

The *Golden Rule*; or, *Rule of Three.*

THIS Rule is the moſt excellent and uſeful Rule of all the Rules in Arithmetick.

It ſhews how by having three Numbers given, to find a fourth, that ſhall bear the ſame Proportion to the Third that the Second doth to the Firſt.

The greateſt Difficulty lies in ſtating your Queſtion, which that you may do, obſerve the following Rule; remember that your firſt and third Numbers be of one Name, and the fourth will be of the ſame Name as the ſecond. Then × the ſecond and third together, and ÷ that Product by the firſt. The Quotient will be the Anſwer.

EXAMPLES.

Queſtion 1ſt. If 100 Pounds in one Year gain 4 Pounds Intereſt, what will 75 Pounds gain in that Time?

Place the Numbers in the following Manner.

```
     l.     l.    l.
If 100  :  4  :  75
                  4
               ----
        1|00)3|00(3 Pounds Anſwer.
             3
            ---
             0
```

100 Pounds the firſt Number is a Principal, and ſo is 75 Pounds the third Number, ſo they are both one Name; 4 Pounds the ſecond Number is an Intereſt, and ſo is 3 Pounds, the Number found, and Anſwer to the Queſtion.

Queſtion 2d. If 1 Acre of Land be worth 15 Shillings a Year, what is 40 Acres (of the like Land) worth by the Year? Anſwer 600 Shillings = 30 Pounds

```
a.    s.     a.    s.
1  :  15  ::  40  :  600
```

Queſtion 3d. Cloth at 5 Shillings *per* Yard, what comes 72 Yards to? *Anſwer* 18 Pounds.

y. s. y. s. l.
For as 1 : 5 :: 72 : 360 = 18

Queſtion 4th. Nails at 5 Pence the Hundred, how many for 3 Pence? *Anſwer* 60. For,

d. ns. d. n.
As 5 : 100 :: 3 : 60.

Queſtion 5th. If one Yard of Cloth coſt 6 *s.* what will 74 Yards of the ſame coſt? *Anſwer* 22 *l.* 4 *s.* For,

y. s. y. s. l. s.
As 1 : 6 :: 74 : 444 = 22 : 4.

Queſtion 6th. If 15 Yards of Cloth coſt 7 *l.* 10 *s.* what comes 7 Packs to, each Pack containing 8 Parcels, and each Parcel 108 Yards at that Rate? *Anſwer* 3024 *l.* Before you ſtate Queſtions of this Nature, reduce them in the following Manner.

7 × 8 = 56 and 56 × 108 = 6048 Yards.

l. s.
And 7 : 10 = 150 Shillings. Then ſay

```
   ys.    s.       ys.
If 15 : 150 :: 6048
                 150
           ---------
              302400
              6048
           ---------
          15)907200(60480 Shillings.  The
              90           Anſwer = 3024 l.
              --
               72
               60
               --
               120
               120
               ---
                 0
```

Queſtion 7th. If 4 lb. of Tea coſt 1 *l.* 13 *s.* 4*d.* what will 4 Baggs coſt, each Bag weighing 2 C. 3 q. 21 lb. at that Rate? When reduced the Stating will ſtand thus:

```
      ℔ : d.  ::  ℔  :  d           l. s. d.
If  4 : 400 :: 1316 : 131600 = 548 : 6 : 8
```

N. B. You may contract Queſtions in this Rule when you can divide the firſt and third Term by one Diviſor, as in this Queſtion, I divide them by 4, and then the Work will ſtand thus.

```
   ℔ : d.  ::  ℔  :  d.          l.  s.  d.
If 1 : 400 :: 329 : 131600 = 548 : 6 : 8
```

as before.

Queſtion 8th. If 108 Pounds in 12 Months gain 6 Pounds Intereſt, how much will it gain in 7 Months? *Anſwer*, 3 *l.* 10 *s.* 0 *d.*

m. : l : m.
For as 12 : 6 :: 7 : 3 *l.* 10 *s.* 0 *d.*

The *Rule of Three Reverſe*

IS that by which we find, from three Numbers given, a fourth, in a reciprocal Proportion, inverted to the Proportion given. Obſerving the Rule given in direct Proportion, it will be needleſs to ſay any thing more of it.

EXAMPLES.

Queſtion 1ſt. If 8 Men do a Piece of Work in 12 Days, how many Men muſt there be to do the ſame in 6 Days? *Anſwer*, 16 Men.

D. : M. : D.
For if 12 : 8 : 6
12

6)96(16 Men the *Anſwer.*
6

36
36

0

In this Queſtion, I conſider that the leſs Time, will require the more Men; ſo I multiply the middle Term by the greater Extream, and divide that Product by the leſs, and the Quotient is the Anſwer. Obſerve this in all the following Queſtions.

Queſtion 2d. If 8 Men do a Piece of Work in 12 Days, in how many Days will 16 Men do the ſame? *Anſwer*, 6 Days.

m. : d. :: m.
For if 8 : 12 :: 16
8
———
16)96(6 Men the *Anſwer*.
96
———
0

Queſtion 3d. How many Yards of Canvas that is Ell wide, will line 20 Yards of Say three Quarters wide. *Anſwer*, 12.

q. y. q. y.
For if 3 : 20 :: 5 : 12; for the wider the Cloth the leſs Number of Yards it requires. So if I × 20 by 3, and ÷ by 5.

Queſtion 4th. How many Foot of Matting of 2 Foot wide, will cover a Floor 20 Foot wide and 24 Foot long? *Anſwer*, 240 Foot.

f.w. : f.l. :: f.w. : f.l.
For if 20 : 24 :: 2 : 240

For the narrower the Matting, the more it will require; ſo I multiply 20 by 24, and divide by 2. The Quotient is the *Anſwer*.

Queſtion 5th. If (according to the Statute) a right angled Paralelogram of 40 Perches in length, and 4 in breadth, make an Acre of Land, how much in length muſt there be, when the breadth is 16 Perches? *Anſwer* 10 Perches.

b. l. b. l.
For if 4 : 40 :: 16 : 10

Queſtion 6th. If 12 Inches in length and 12 in breadth make a ſuperficial Foot, how much muſt there be in length when the breath is 4 Inches? *Anſwer*, 36 Inches long.

i.l. i.b. i.b. i.l.
For if 12 : 12 : :: 4 : 36

Queſtion 7th. If one lend a Friend 640 Pounds 8 Months, what Sum muſt he lend 12 Months to retaliate the former Favour? *Anſwer*, 426l. 13s. 4d.

m. l. m.
For if 8 : 640 :: 12 : 426l. 13s. 4d.

Queſtion 8th. If one lends his Friend 600 Pounds 8 Months, how long muſt he lend 400 Pounds to recompence the former Kindneſs? *Anſwer*, 12 Months.

l. m. l. m.
For if 600 : 8 :: 400 : 12

Queſtion 9th. If I have 1200 ℔. Weight carried 36 Miles for 24 Shillings, how many Miles ſhall 1800 ℔. Weight be carried for the ſame Sum. *Anſwer*, 24 Miles.

℔. m. ℔. m.
For if 1200 : 36 :: 1800 : 24.

Queſtion 10th. If for 24 Shillings, I have 1200 ℔. Weight carried 36 Miles, how many Pound Weight ſhall I have carried 24 Miles for the ſame Money? *Anſwer*, 1800 ℔. Weight.

m. ℔. m. ℔.
For If 36 : 1200 :: 24 : 1800

Queſtion 11th. A General is beſieged in a Town, in which are 1200 Soldiers, with Proviſion for 3 Months; how many Men muſt he diſmiſs, that his Proviſion may ſerve thoſe that remains 8 Months? He muſt keep 450, and diſmiſs 750.

m. s. m. s.
For if 3 : 1200 :: 8 : 450

Queſtion 12th. If a Field will graze 21 Horſes 6 Weeks, how many will it graze 7 Weeks? *Anſwer*, 18 Horſes.

w. h. w. h.
For if 6 : 21 :: 7 : 18

Queſtion 13th. If a Field will graze 21 Horſes for 6 Weeks, how long will it graze 18 Horſes? *Anſwer*, 7 Weeks.

h. w. h. w.
For if 21 : 6 :: 18 : 7

Queſtion 14th. If 136 Maſons build a Fort in 28 Days, to preſerve the Soldiers from the Enemy, but the General would have it built in 8 Days, how many Men muſt be ſet to Work? *Anſwer*, 476 Maſons.

d. m. d. m.
For if 28 : 136 :: 8 : 476

The

The Double Rule of Three; or, Rule compoſed of five Numbers.

I Shall give no Directions about placing the Numbers, more than what has been delivered in the direct and reverſe Rules, only I ſhall make two ſingle Statings, and work the Queſtion as if it had but one Stating, by the help of the Letter X; the uſe of which will be better underſtood by an Example than a great many Words.

Queſtion 1ſt. If 100 l. Principal in 12 Months, gain 6 l. Intereſt, what will 75 l. Principal gain in 9 Months? *Anſwer* 3 l. 7 s. 6.

$$\left.\begin{array}{l}\text{If } 100 : 6 :: 75 : X \\ \text{If } 12 : X : 9\end{array}\right\}\text{Theſe are Direct.}$$

100 × 12 = 1200 the Diviſor.
And 75 × 6 = 450 × 9 = 4050 the Dividend.
And 4050 ÷ by 1200 gives 3l. 7s. 6d. the *Anſwer*.

N. B. The X always ſupplies the fourth Place in the firſt Stating, and the ſecond Place in the ſecond Stating; and is ſuppoſed to be of the ſame Name as the Anſwer is of.

Queſtion 2d. If 100 l. Principal, in 12 Month gain 6 l. in how long Time will 75 l. gain 3 l. 7 s. 6 d.? *Anſwer*, 9 Months.

Reduce 6 l. and 3 l. 7 s. 6 d. into Pence, and the Statings will ſtand thus.

If 1440 : 11 :: 810 : X. This is Direct.
If 100 : X :: 75 This is Reverſe.
1440 × 75 = 108000 The Diviſor.

And 810 × 12 = 9720 × 100 = 972000 the Dividend. Then 972000 ÷ 108000 gives 9 in Quotient the *Anſwer*.

Queſtion 3d. If 100 l. Principal, in 12 Months gain 6 Pounds, what Principal will gain 3 l. 7. 6 d. in 9 Months? *Anſwer*, 75 l.

Reduce the Intereſts, and it will ſtand thus.
If 1440 : 100 :: 810 : X. This is Direct.
If 12 : X : 9 This is Reverſe.
Then 1440 × 9 = 12960 the Diviſor.

And 810 × 100 = 81000 × 12 = 972000 the Dividend. And 972000 ÷ 12960 gives 75 in the Quotient the *Anſwer*. Theſe three Queſtions mutually prove one another.

Queſtion 4th. If the Carriage of 100 ℔ Weight 30 Miles coſt 1 Shilling, what will the Carriage of 500 ℔ Weight coſt, being carried 100 Miles?

s.
If 100 : 1 :: 500 : X. This is Direct.
If 30 : X :: 100 This is Direct.
Then 100 × 30 = 3000 The Diviſor.
And 500 × 1 = 500 × 100 = 50000 the Divid.
And 50000 ÷ 3000 = 16 s. 8 d. the *Anſwer*.

Queſtion 5th. If a Regiment of 936 Soldiers eat 351 Quarters of Wheat in 168 Days, how many Quarters will ſuffice an Army of 11232 Soldiers 56 Days?

If $936 : 351 :: 11232 : X$. This is Direct.
If $168 : X :: 56$ This is Direct.
Then $936 \times 168 = 157248$ the Diviſor.
And $11232 \times 351 = 3942432 \times 56 = 220776192$ the Dividend.

And $220776192 \div 157248$ gives 1404 Quarters the *Anſwer.*

Queſtion 6th. If a Regiment of 936 Soldiers eat 351 Quarters of Wheat in 168 Days, how many Soldiers will 1404 Quarters ſuffice 56 Days?

If $351 : 936 :: 1404 : X$ This is Direct.
If $168 : X :: 56$ This is Reverſe.
Then $551 \times 56 = 19656$ The Diviſor.
And $1404 \times 936 = 1314144 \times 168 = 220776192$ the Dividend.

And $220776192 \div 19656$ gives 11232 the Soldiers it will ſuffice the *Anſwer.*

Queſtion 7th. If a Regiment of 936 Soldiers eat 351 Quarters of Wheat in 168 Days, how many Days will 1404 Quarters ſuffice an Army of 11232 Soldiers?

If $351 : 168 :: 1404 : X$ This is Direct.
If $936 : X :: 11232$ This is Reverſe.
Then $11232 \times 351 = 3942432$ The Diviſor.
And $1404 \times 168 = 235872 \times 936 = 220776192$ the Dividend.

And $220776192 \div 3942432$ gives 56 Days the *Anſwer.*

Queſtion 8th. If 8 Men in 7 Days mow 40 Acres, how many Acres will 24 Men mow in 28 Days?

If 8 : 40 :: 24 : X This is Direct.
If 7 : X :: 28 This is Direct.
Then 8×7=56 the Diviſor.
And 24×40=960×28=26880 the Dividend.
And 26880÷56 gives 480 Acres the *Anſwer*.

Queſtion 9th. If 8 Men in 7 Days mow 40 Acres, in how many Days will 480 Acres be mowed by 24 Men?

If 8 : 7 :: 24 : X This is Reverſe.
If 40 : X :: 480 This is Direct.
Then 24×40=960 The Diviſor.
And 480×8=3840×7=26880 The Dividend.
Then 26880÷960 gives 28 The *Anſwer*.

Queſtion 10th. If 8 Men in 7 Days mow 40 Acres, how many Men muſt there be to mow 480 Acres in 28 Days?

If 7 : 8 :: 28 : X This is Reverſe.
If 40 : X :: 480 This is Direct.
Then 28×40=1120 The Diviſor.
And 480×8=3840×7=26880 The Dividend.
Then 26880÷1120=24 Men the *Anſwer*.

Queſtion 11th. If 40 Shillings be the Wages of 8 Men for 5 Days, what will be the Wages of 32 Men for 24 Days?

If 8 : 40 :: 32 : X This is Direct.
If 5 : X :: 24 This is Direct.
Then 5×8=40 the Diviſor.
And 32×40=1280×24=30720 The Dividend.
Then 30720÷40=768 Shillings=38l. 8s. The *Anſwer*.

Queſtion 12th. If 14 Horſes eat 56 Buſhels of Provender in 16 Days, how many Buſhels will 20 Horſes eat in 24 Days?

If 14 : 56 :: 20 : X This is Direct.
If 16 : X :: 24 This is Direct.
Then $16\times14=224$ the Diviſor.
And $56\times20=1120\times24=26880$ The Dividend.
Then $26880\div224$ gives 120 Buſhels the *Anſwer*.

Queſtion 13th. If 8 Cannons in one Day ſpend 48 Barrels of Powder, how many Barrels will 24 Cannons ſpend in 12 Days?

If 8 : 48 :: 24 : X This is Direct.
If 1 : X :: 12 This is Direct.
Then $1\times1=8$ The Diviſor.
And $24\times12=288\times48=13824$ The Dividend.
Then $13824\div8$ gives 1728 Barrels the *Anſwer*.

Queſtion 14th. If in a Family conſiſting of 7 Perſons, there are drank out 36 Gallons of Beer in 12 Days, how many Gallons will there be drank out in 8 Days by another Family, conſiſting of 14 Perſons?

If 7 : 36 :: 14 : X This is Direct.
If 12 : X :: 8 This is Direct.
Then this $12\times7=84$ The Diviſor.
Then $14\times8=112\times36=4032$ The Dividend.
And $4032\div84$ gives 48 Gallons the *Anſwer*.

Queſtion 15th. If, when the Buſhel of Wheat coſts 3s. 4d. the Penny Loaf weighs 12 Ounces, what will a Loaf worth 9 Pence weigh, when Wheat is 10 Shillings the Buſhel?

If 40 : 12 :: 120 : X This is Reverſe.
If 1 : X :: 9 This is Direct.
Then 120×1=120 The Diviſor.
And 40×12=480×9=4320 The Dividend.
And 4320÷120=36 Ounces the *Anſwer.*

Queſtion 16th. A Man put out 75 l. to receive Intereſt for the ſame, and when it had continued 9 Months, he received (for Principal and Intereſt) 78 l. 7 s. 6 d.; now at what Rate per Cent. *per Annum* did he receive Intereſt?

If 75 : 810 :: 100 : X This is Direct.
If 9 : X : 12 This is Direct.
Then 75×9=675 The Diviſor.
And 810×100=81000×12=972000 Dividend.
Then 972000÷675 gives 1440 Pence=6 Pound the *Anſwer.*

Queſtion 17th. If a Captain ſet 300 Pioneers to work, and they in 8 Hours caſt a Trench of 200 Rods, how many Pioneers will be able, with a like Trench in 3 Hours, to intrench a Camp 34000 Rods?

If 200 : 300 :: 3400 : X This is Direct.
If 8 : X :: 3 This is Reverſe.
This 200×3=600 The Diviſor.
And 3400×300=1020000×8=8160000 The Dividend.
Then 8160000 ÷ 600 gives 13600 Pioneers the *Anſwer.*

Queſtion 18th. If 20 Dogs for 30 Groats go 40 Weeks to Graſs, how many Hounds for 60 Crowns may winter in that Place? *N. B.* Twelve Weeks is a Winter.

C. D. C.
If 2 : 20 : : 60 : X This is Direct.
W. D. W.
If 40 : X : : 12 This is Reverſe.
Then $12\times2=24$ The Diviſor.
And $20\times60=1200\times40=48000$ The Dividend.
Then $48000\div24$ gives 2000 Dogs the *Anſwer*.

Queſtion 19th. If 35 Ells of Vienna make 24 at Lyons, and 3 Ells at Lyons make 5 Ells at Antwerp, and 100 Ells at Antwerp 125 at Franckfort, how many Ells at Franckfort make 42 at Vienna? *Anſwer* 60 Ells.

$$\left.\begin{array}{l}\text{If } 100 : 125 :: 5 : X \\ \text{If } 3 : X :: 24 : X \\ \text{If } 35 : X :: 42\end{array}\right\} \text{ Theſe are all Direct.}$$

Then $100\times3=300\times35=10500$ The Diviſor.
And $125\times5=625\times24=15000\times42=630000$ The Dividend.

Then $630000\div10500$ gives 60 Ells the *Anſwer*.

Notation of Vulgar Fractions.

A Fraction is properly an Unit divided into Parts, and is noted by two Numbers thus $\frac{7}{12}$, the lower denoteth the Number of Parts the Unit is divided into, and is called the Denominator; the upper ſhews how many of theſe Parts are ſignified, and is called the Numerator.

As if the Integer be one Shilling broken into Twelve Parts, then this Fraction $\frac{7}{12}$, is in Value 7 Pence, and is read ſeven Twelfths.

Fractions are of three Kinds, *viz.*

This $\frac{7}{12}$ is a ſimple Fraction; the Numerator is the leaſt. This $\frac{1}{2}$ of $\frac{3}{4}$ is a Compound Fraction, or a Part of a Part. This $\frac{7}{4}$ is an improper Fraction; the Numerator is the largeſt.

N. B. If the Numerator be equal to, or greater than the Denominator, it is an improper Fraction. Thus $\frac{4}{4}=1$ and $\frac{7}{4}=1$ and $\frac{3}{4}$.

Reduction of Vulgar Fractions

TEacheth to bring Fractions of different Denominators, into one common Denominator. To bring a Compound Fraction to a Simple one; and to expreſs a Number like an improper Fraction.

To bring Simple Fraction into one common Denominator. *Rule*, × all the Denominators into one another for a common Denominator; and every Numerator into all the Denominators, but its own, for new Numerators.

EXAMPLES.

Reduce $\frac{2}{3}$. $\frac{3}{4}$. $\frac{5}{6}$. and $\frac{1}{2}$ into a Common Numerator.

Firſt $3 \times 4 \times 6 \times 2 = 144$ the Common Denominator.
Then $2 \times 4 \times 6 \times 2 = 96$ New Numerator $= \frac{2}{3}$.
And $3 \times 3 \times 6 \times 2 = 108$ New Numerator $= \frac{3}{4}$.
And $5 \times 4 \times 3 \times 2 = 120$ New Numerator $= \frac{5}{6}$.
And $1 \times 6 \times 4 \times 3 = 72$ New Numerator $= \frac{1}{2}$.

The New Fractions will ſtand thus:

$$\begin{array}{ccccl} 96 & 108 & 120 & 72 & \text{New Numerators.} \\ \hline 144 & 144 & 144 & 144 & \text{Common Denominator.} \end{array}$$

N. B. If I had divided my Common Denominator by the Denominator of each Fraction, and × that Quotient by the Numerator of the Fraction, the Product would be a new Numerator, as before.

By either of theſe Ways you may reduce Simple Fractions to one Common Denominator.

To reduce Compound Fractions to Simple ones. *Rule*, × all the Numerators together for a new one, and × all the Denominators together for a new one.

EXAMPLE.

EXAMPLE.

Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{1}{2}$ to a Simple Fraction.
Firſt $2\times3\times1=6$ the Numerator.
And $3\times4\times2=24$ the Denominator.
Then the Simple Fraction will be $\frac{6}{24}$.

To reduce a mixt Number to an improper Fraction. *Rule*, × the whole Number by the Denominator of the Fraction, and to the Product add the Numerator of the Fraction; and under this Sum place the Denominator of the Fraction, and it is done.

EXAMPLE.

Reduce $4\frac{3}{4}$ to an improper Fraction.
$4\times4=16+3=19$. Then the Fraction is $\frac{19}{4}$.

Expreſs 7 like an improper Fraction.
Then it is $\frac{7}{1}$, or $\frac{28}{4}$, or $\frac{21}{3}$, or $\frac{63}{9}$, &c. Any of theſe Fractions are in Value 7.

To reduce an improper Fraction to a mixt Number. *Rule*, Divide the Numerator by the Denominator, and the Remainder place over the Diviſor, and its done.

Thus $\frac{19}{4} = 4\frac{3}{4}$.

EXAMPLE.

To bring a Fraction to its loweſt Terms. *Rule*, Divide the Denominator by the Numerator, and if any thing remain, divide your laſt Diviſor by it, ſo every Diviſor, if there is a Remainder, becomes the Dividend, and the Remainder your Diviſor; and the laſt Diviſor, if there be no Remainder, is the

Number that will reduce the Fraction to its loweſt Term. If there is no ſuch Diviſor, the Fraction is in its loweſt Terms.

EXAMPLE.

Reduce $\frac{1008}{1512}$ into its loweſt Term. Place it thus:

```
1008)1512(1
     1008
     ————
      504)1008(2
          1008
          ————
            0
```

Now 504, which was my firſt Remainder and laſt Diviſor, will divide both the Numerator and Denominator of the given Fraction, and ſo reduce it to $\frac{2}{3}=\frac{1008}{1512}$.

Reduce $\frac{2}{3}$ and 4 $\frac{3}{7}$ and $\frac{1}{2}$ of $\frac{3}{4}$, to a Common Denominator.

The mixt Number reduced to an improper Fraction, is $\frac{31}{7}$.

The Compound Fraction reduced to a Simple one, is $\frac{3}{8}$.

Then for Reduction, they will ſtand in the following Manner:

$\frac{2}{3}$. $\frac{31}{7}$ and $\frac{3}{8}$.

Then $3\times7\times8=168$ the Common Denominator.
And $2\times7\times8=112$ the New Numerator$=\frac{2}{3}$.
And $31\times3\times8=744$ the New Numerator$=\frac{31}{7}$.
And $3\times7\times3=63$ the New Numerator$=\frac{3}{8}$.
And they will ſtand thus $\frac{112}{168}$ $\frac{744}{168}$ $\frac{63}{168}$.

To reduce a Fraction of one Denomination to another.

What part of a Pound is $\frac{3}{4}$ of a Penny; it will be this Compound Fraction, $\frac{3}{4}$ of $\frac{1}{12}$ of $\frac{1}{20} = \frac{3}{960}$

To bring a Fraction from a greater to a leſs Denomination. *Rule*, multiply the Numerator by the Parts contained betwixt it, and what you would reduce it to.

EXAMPLE.

Reduce $\frac{2}{3}$ of a Pound to the Fraction of a Penny

$2 \times 12 \times 20 = 480$ then it is ^{480}

To find the Value of any Fraction in the known Parts of Coin, Weight, Meaſure, &c. *Rule.* Multiply the Numerator of the Fraction by the next inferior Denomination, that is equal to a Unit of the ſame Denomination with the Fraction, and divide that Product by the Denominator of the given Fraction, and the Quotient is its Value, in the Parts you multiplied by; and if there be a Remainder after your Diviſion, multiply by the next Inferior, and divide as before, and ſo continue to do till you can bring it no lower.

What is the Value of $\frac{133}{480}$ of a Pound Sterling?
Anſwer 5 s. 6 d. $\frac{1}{2}$

```
      133
       20
    ------
480)2660(5
    2400
    ------
     260
      12
    ------
     520
    260
482)3120(6
    2880
    ------
     240
       4
    ------
480)960(2
    960
    ------
      0
```

Addition of Vulgar Fractions.

R U L E.

BRING the Fractions to be added into one Denominator by the Rules in Reduction; then add their Numerators, and under the Sum put the Common Denominator, and it is done.

EXAMPLE.

Add $\frac{2}{3}$ $\frac{3}{4}$ and $\frac{5}{6}$ of a Pound into one Sum.
$3\times4\times6=72$, the Common Denominator.
The Numerators will be found 48, 54, and 60.
Then $48+54+60=162$. The Sum is $\frac{162}{72}$.

Subſtraction of Vulgar Fractions.

RULE.

BRING the Fractions to one Denominator, and ſubtract the Numerators one from another, and under place the Common Denominator, and it is done.

EXAMPLE.

From $\frac{3}{4}$ take $\frac{2}{8}$. Firſt $8\times4=32$ Common Denominator.

And $3\times8=24$ New Numerator. And $2\times4=8$. Then $\frac{24}{32}-\frac{8}{32}=\frac{16}{32}$.

Multiplication of Vulgar Fractions.

RULE.

REduce mixt Numbers into improper Fractions, expreſs whole Numbers like Fractions, and bring Compound Fractions into Simple. Then multiply the Numerators together for a new Numerator, and the Denominators together for a new Denominator; which Numerator and Denominator is the Product ſought.

EXAMPLE.

Multiply 2s. 6d. by 2s. 6d. as 2s. 6d.$=\frac{1}{8}$.
Then $\frac{1}{8}\times\frac{1}{8}=\frac{1}{64}=$0l. 0s. 3d.$\frac{3}{4}$. And $\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$.

Diviſion of Vulgar Fractions.

REduce the Fractions by the Rule in Multiplication, then invert your Diviſor, and multiply then as in Multiplication, and the Product will be your Quotient.

EXAMPLE.

Divide $\frac{8}{117}$ by $\frac{4}{13}$, the Diviſor invert, it will ſtand thus: $\frac{13}{4})\frac{8}{117}(\frac{104}{468}$, the Quotient; for $13\times8=104$. And $117\times4=468$.

Divide $\frac{36}{114}$ by $\frac{6}{19}$. Invert the Diviſor, and it will ſtand thus: $\frac{19}{6})\frac{36}{114}(\frac{684}{684}=1$.

Arithmetical Progreſſion

IS when a Rank of Numbers above two increaſe or decreaſe by the continual Addition or Subtraction of ſome equal Number. So 2, 7, 12, 17, 22, 27, 32, and 36, 32, 28, 24, 20, 16, 12, 8, 4, are two Ranks of Numbers in Arithmetical Progreſſion; the firſt increaſing by the continual Addition of 5, and the ſecond decreaſing by the continual Subtraction of 4, and ſo of any other. In this Progreſſion five Things are to be conſidered, *viz.*

1ſt. The firſt Term.
2d. The laſt Term.
3d. The Number of Terms.
4th. The common Difference.
5th. The Sum of all the Terms.

Any of theſe being given, the other may be found, and admits of 20 Varieties. See *Oughtred*'s Key of the Mathematicks, Page 85.

Theorem 1ſt. The laſt Term is always = to the firſt, and ſo many common Differences as there are Number of Terms leſs by one, as in this Progreſſion, 4, 9, 14, 19, 24, 29. Here the Number of Terms is 6, and the common Difference is 5, and the firſt Term is 4. Then $6-1=5$ the Number of Terms leſs one. And $5\times5=25+4=29$ the laſt Term.

Theorem 2d. If the firſt Term is added to the laſt, and that Sum × by the Number of Terms, the Product will be double the Sum of the whole Progreſſion.

As in the laſt, $4+29=33$.
And $33\times6=198$ double the Sum of the whole.
Then $198\div2=99$ the Sum of the Progreſſion.

Queſtion 1ſt. A Merchant bought 60 Bales of Holland, and agreed to pay for the firſt Bale 4l. for the ſecond 7l. ſo increaſing the Price of every Bale by 3l. what did he pay for the laſt Bale, and what did the whole coſt?

Here is given 4 the firſt Term, and 3 the common Difference, and 60 the Number of Terms. Then $59\times3=177$; to this add the firſt Term, and it will be $177+4=181$ l. the Price of the laſt Bale. Then $181+4=185$, and $185\times60=11100$, this $\div 2=5550$ l. the Price of the whole.

If 100 Eggs are placed in a right Line, a Yard diſtant from one another, and the firſt a Yard diſtant from a Baſket, how far muſt one go before he brings the Eggs one by one into the Baſket.

N. B. They muſt begin at the Baſket.

2 Firſt Term.
200 Laſt Term.

202 The Sum of the firſt and laſt Term.
100 The Numbers of Terms.

20200 Double the Yards gone. The half is

10100 Yards gone in the whole = To 5 Miles and 1300 Yards.

Theorem 3d. If double the Sum be divided by the Sum of the firſt and laſt Terms, the Quotient will be the Number of Terms.

Theorem 4th. If the firſt Term be ſubtracted from the laſt, and the Remainder divided by the Number of Terms leſs by one; the Quotient will be the common Difference.

A Gentleman ordered his Steward to diſburſe 1700 Pounds amongſt his Tenants, that had ſuffered by the Contagion amongſt the Cattle, and bring him an Account of the Number of Sufferers, and how much each received of his Bounty; but the Steward loſt the Account, and only remembers that the leaſt Sufferer had 3l. and the greateſt 133l. and that the Difference was equal betwixt every two next. How many Tenants received of the Bounty, and what was the Difference?

The laſt Term 133
The firſt Term 3

Their Sum is 136 the Diviſor.

And 1700×2=3400 the Dividend.

Then 3400 ÷ 136 = 25 the Number of Tenants.

And 133−3=130l. and this divided by 24, gives 5l. 8s. 4d. the Difference betwixt every two next. So that the leaſt received 3l. the next 8l. 8s. 4d. the third 13l. 16s. 8d. &c.

Geometrical Progreſſion

IS when Numbers increaſe by a like Proportion, as 4, 8, 16, 32, 64, 128, are in Geometrical Progreſſion, for the ſecond contains the firſt as many Times as the third contains the ſecond, and ſo the fourth the third, &c.

In this Progreſſion, the ſame Things are to be conſidered as in Arithmetical Progreſſion; as the firſt Term, the laſt Term, the Number of Terms, the Ratio, or common Exceſs; and the Sum of all the Terms.

If over a Series of Terms in Geometrical Progreſſion, be ſet for Indices a Series of Terms in Arithmetical Progreſſion, to every four Numbers taken in Arithmetical Progreſſion, will anſwer four Numbers geometrically proportional.

Examples.

EXAMPLES.

Indices	0.	1.	2.	3.	4.	5.
Terms	4.	8.	16.	32.	64.	128.

Theſe Indices, or Exponents, beginning with a Cypher, and placed over any Series in Geometrical Progreſſion, ſhews the Diſtance of any Term from the firſt.

Theorem 1ſt. In any Geometrical Progreſſion, if you add the Exponents of any two Terms, and multiply the Terms together, and divide the Product by the firſt Term, the Quotient will be the Term anſwering the Sum of the Indices you added.

EXAMPLES.

0.	1.	2.	3.	4.	5.	6.
4.	8.	16.	32.	64.	128.	256.

In this Progreſſion, if the third and fourth Terms be × together, and the Product divided by the firſt, the Quotient will be the ſeventh Term, and ſo of any other.

That is, $64 \times 32 = 2048 \div 4 = 512$ the ſeventh Term.

Theorem 2d. If the laſt Term be × by the Ratio, and from the Product ſubtract the firſt Term, and divide the Remainder by one leſs than the Ratio, the Quotient will be the Sum of the whole Progreſſion.

A Merchant has 12 Yards of Velvet, which he values at 1l. 4s. per Yard; but he ſells it for 1s. the firſt Yard, 2s. the ſecond Yard, 4s. the third Yard, &c. doubling the Price of each Yard to the twelfth and laſt Yard. The Queſtion is, what does he gain or loſe?

Indices	0.	1.	2.	3.	4.	5.	6.
Terms	1.	2.	4.	8.	16.	32.	64.

Then $64 \times 32 = 2048$ Price of the laſt Yard.

For the Indices $5 + 6 = 11$ the Diſtance of that Term from the firſt.

And $2048 \times 2 = 4096 - 1 = 4095$ Shillings; the Value of the 12 Yards at the Price agreed upon = 204l. 15s. which is 17l. 1s. 3d. *per* Yard; ſo that 17l. 1s. 3d. — 1l. 4s. 0d. = 15 : 17 : 3 what the Merchant gains *per* Yard above the Value.

Suppoſe an Eſtate of 72 l. *per* Annum was ſold on theſe Conditions; the firſt of *January* the Buyer is to pay 3 s. the firſt of *February* 9 s. the firſt of *March* 27s. and ſo on in Geometrical Proportion, to the laſt and 12th Payment on the firſt of *December*. The Queſtion is, to know what the 12th Payment was, and what the Eſtate came to?

Indices	0.	1.	2.	3.	4.	5.	6.
Terms	3.	9.	27.	81.	243.	729.	2187.

Then $2187 \times 729 = 1594323$, this $\div$ by three gives $531441 =$ the laſt Payment in Shillings.

Then $531441 \times 3 = 1594323 - 3 = 1594320$.

This $1594320 \div 2 = 797160$ Shillings the Eſtate coſt $= 39858$ Pounds the *Anſwer*.

If a Horſe, having 4 Shoes, and 6 Nails in each Shoe, is ſold on this Condition, that the firſt Nail is to be a Halfpenny, the ſecond a Penny, the third Two-pence, &c. doubling every Nail; how much would the Price of the Horſe come to?

Indices	0.	1.	2.	3.	4	5.	6.	7.
Terms	1.	2.	4.	8.	16.	32.	64.	128.

Firſt, 128×128=16384 Price of the 15th Nail.

And, 16384 × 64=1048576 Price of the 21ſt Nail.

Then, 1048576 × 8 = 8388608 Halfpence the Price of the 24th Nail.

And, 8388608×2= 16777216−1=16777215 The Price of the Horſe in Halfpence, equal to 34952l. 10s. 7d. ½.

Variation of Quantities

SHEWS, how many ſeveral Ways any Number of Qualities may be changed or varied, with reſpect to their Places.

EXAMPLE.

How many Changes may be rung on 8 Bells? *Rule.* × 1, 2, 3, 4, 5, 6, 7, 8, into one another, and the laſt Product is the *Anſwer.*

1×2×3×4×5×6×7×8=40320 The *Anſwer*, and ſo many Changes may be rung on 8 Bells.

How many different Poſitions may a Captain order 12 Men in?

1×2×3×4×5×6×7×8×9×10×11×12= 479001600 The *Anſwer*, and ſo many different Poſitions may they be placed in with regard to one another; which is almoſt incredible.

Notation of Decimal Fractions.

A Decimal Fraction is ſuch, whoſe Denominator is not expreſſed, but underſtood; and is a Unit with as many Cyphers annexed, as there is Places in the Numerator.

So $\frac{5}{10}$ will be expreſſed thus, ,5; and $\frac{25}{100}$ thus, ,25; and $\frac{425}{1000}$ thus, ,425; and this Point, or Comma is prefixed, to diſtinguiſh them from an Integer.

N. B. A Cypher placed to the left Hand of an Integer, and to the right Hand of a Decimal, neither increaſeth nor decreaſeth the Value, as 4. 04. 004. Theſe being Integers, do ſtill retain the ſame Value.

And ,7. ,70. ,700. theſe being Decimals, do retain the ſame Value, every one being ,7 Parts, when the Integer is divided into 10 Parts.

But Cyphers placed to the right Hand of an Integer, increaſeth its Value; and to the left Hand of Decimal, decreaſeth its Value.

As ,5 is five Tenth Parts.
,05 is five Hundred Parts.
,005 is five Thouſand Parts.
,0005 is five Ten thouſand Parts.

By this you may obſerve, Decimal Fractions do decreaſe in a Ten-fold Proportion to the right Hand.

Reduction of Decimals.

BY this we find the Decimal, equal in Value to any Vulgar Fraction given; and on the contrary, find the Value of a Decimal Fraction in the known Parts of Coin, Weight, Meaſure, &c.

To bring a Vulgar Fraction to a Decimal. *Rule.* Add as many Cyphers as you pleaſe to the Numerator of the given Fraction; and then divide by the Denominator, and the Quotient is the Decimal required. Diſtinguiſh the Cyphers you add with a Comma.

EXAMPLES.

Reduce $\frac{3}{4}$ to a Decimal Fraction.

```
4)3,00(,75 The Decimal.
  2 8
 ———
   20
   20
```

Reduce $\frac{1}{12}$ to a Decimal Fraction.

```
12)1,0000(,0833
    96
    ——
     40
     36
     ——
      40
      36
      ——
       4
```

N. B. In the above Example, I firſt ſay how many Times 12 the Denominator, in 1 the Numerator; and then I make the Comma in the Quotient to diſtinguiſh it, and then I ſay how many times 12 in 10, and I place a Cypher after the Comma; and ſo you muſt take your Cyphers one by one, placing Cyphers in your Quotient, if you cannot have the Diviſor in your Dividend. And if you annex a thouſand Cyphers, there would be the ſame Remainder, and conſtantly 3's in your Quotient; and they are called Repetends, or repeating Numbers. But if we bring Decimals to 5 or 6 Places, it will be exact enough in moſt Caſes, and the Remainder will be of no Value.

Reduce Nine Pence into a Decimal, a Pound being the Integer, 240 Pence being a Pound, it will be $\frac{9}{240}$; then add Cyphers to the Numerator, and divide as before.

```
240)9,0000(,0375
     720
     ----
     1800
     1680
     -----
      1200
      1200
      -----
         0
```

Reduce 15 Shillings into the Decimal of a Pound.

20)15,00(,75 = 15 Shillings.
140
100
100
0

The Decimal of any Number of Shillings is found by halving them, and placing a Comma before the Half.

What is the Decimal of 4s. 6d. a Pound the Integer ?

4s. 6d.
12
54

240)54,000(,225=4 Shillings and 6 Pence.
480
600
480
1200
1200
0

What is the Decimal of 12 Penny Weight, a Pound Troy being the Integer? 240 Penny Weight is one Pound. Then it will be $\frac{12}{240}$.

240)12,00(,05=12 Penny Weight.
1200
——
0

What is the Decimal of one Inch, a Foot being the Integer?

12(1,0000(,0833
96
——
40
36
——
40
36
——
0

In this Manner is found the Decimal of any Vulgar Fraction whatſoever. I ſhall omit placing the Tables, becauſe of the eaſy finding the Decimal by the Rule delivered.

To find the Value of any Decimal Fraction in the known Parts of Coin, Weight, Meaſure, Time, Motion, &c.

Rule. Multiply the Decimal given by the Number of Parts of the next inferior Denomination, cutting off as many Figures from the Product, as there is in the Decimal given, and the Remainder, (if any) multiply by the Parts of the next Inferior, cutting off as before. Thus muſt you do, till the Decimal is brought into its leaſt Parts. The Value is what you cut off to the left Hand.

Example.

EXAMPLE.

What is the Value of ,875 of a Pound Sterling?
20 Shillings in a Pound.

17,500
12

1000
500

6,000

So that the Value is 17 s. and 6 d.

What's the Value of ,696875 of a Pound Sterling?

,696875
20

Shillings 13,937500
12

1875000
937500

Pence 11,250000
4

Farthings 1,000000

What is the Value of ,72065 of a Pound Sterling?

,72065
20

Shillings 14,41300
12

82600
41300

Pence 4,95600
4

Farthings 3,82400 the Remain leſs than a Farthing, and may be rejected.

What is the Value of ,6725 of a Hundred Weight?

,6725
4

Quarters 2,6900
28 Pounds in a Quarter.

55200
13800

Pounds 19,3200
16 Ounces in a Pound.

19200
3200

Ounces 5,1200

Addition of Decimals

IS not much different from Addition of Integers; only mind to keep Units under Units in Integers, and Tenths under Tenths in Decimal Parts.

EXAMPLE.

Add ,7435 of a Pound to ,75 of a Pound. Place them in the following Manner.

,7435
75

1,4935

N. B. When you have added your Decimals together, ſo many muſt be cut off with a Comma, as that Decimal Number conſiſts of, which in your Example contains the moſt Places. The reſt, (if any are Integers.

	£.		£.
	,374		42,743
	,067		61,145
	,5746		37,567
	,35		27,915
Sum	1,3656	Sum	169,370

But if the Numbers given to be added are not all of the ſame Denominations, they muſt be brought into Fractions of like Denominations, by finding the Value of the leaſt, and then bringing it by Reduction, to be of the ſame Denomination with the greater.

EXAMPLE.

Add ,375 of a Pound to ,75 of a Shilling.

The ,75 of a Shilling will be found ,0375 of a Pound.

And the Addition will ſtand as follows.

,375
,0375

The Sum ,4125

Subtraction of Decimals

Differs nothing from Subtraction of whole Numbers; only in placing them, mind to place Units under Units in Integers; and Tenths under Tenths, in Decimal Parts.

EXAMPLES.

Subtract ,6725 from ,8765. Place them thus.

From ,8765
Take ,6725

Remains ,2040

So if from 27,625
You ſubtract 8,875

There will remain 18,750

If the Decimal Parts in either Number, have fewer Places than the other, the Vacancy muſt be ſupplied, by annexing ſo many Cyphers as will make them equal, or ſuppoſe them annexed.

EXAMPLE.

From	274,4500	From	274,45
Taſte	89,7425	Take	89,7425
Remains	184,7075	Exceſs	184,7075

But if your Numbers given to be ſubtracted, are not of the ſame Denomination, you muſt, as in Addition, bring them into one Denomination.

EXAMPLE

Subtract ,375 of a Shilling, from ,25 of a Pound. It will ſtand thus;

	£.		
	,25		
	01875		
		s.	d.
Remains	,23125 =	4	$7\frac{1}{2}$.

Or if you bring the Decimal of a Pound to that of a Shilling, the Work will ſtand thus.

From	5,000		
Take	,375		
	=	s.	d.
	4,625 =	4	$7\frac{1}{2}$.

Multiplication of Decimals

IS the ſame as Multiplication of whole Numbers, only mind to cut off with a Comma from the Product, as many Places as you have Decimals in both your Multiplicand and Multiplier; and if there is not ſo many, add Cyphers to the left Hand of the Product.

EXAMPLE.

Multiply ,375
By 2,5
1875
750
,9375 The Product.

Multiply ,652
By ,25
3260
1304
,16300 The Product.

If a Board is 7 Foot 9 Inches long, and 4 Foot 6 Inches broad, what is the ſuperficial Content?

The Inches being reduced to the Decimal of a Foot, the Work will ſtand in the following Manner:

F.
7.75 Length.
4,5 Broad.

3875
3100

34,875

The ſuperficial Content = 34 : 10 $\frac{1}{2}$ (F. : I.)

By Croſs Multiplication, the Work will ſtand thus:

F. I.
7 : 9
4 : 6

3 : 10 : 6
31 : 0

34 : 10 : 6 The Anſwer as before.

Multiply 2 s. 6d. by 2 s. 6d. a Pound being the Integer.

Two Shillings and ſix Pence being reduced to the Decimal of a Pound, the Work will ſtand thus:

,125
,125

625
250
125

,015625=0l. 0s. 3d. $\frac{3}{4}$

This Queſtion performed by Vulgar Fractions, 2 s. 6 d. is $\frac{1}{8}$ of a Pound. Then $\frac{1}{8} \times \frac{1}{8} = \frac{1}{64}$, which valued by the Rule given in Vulgar Fractions, is 0 l. 0 s. 3 d. $\frac{3}{4}$ as before.

Multiply 2 s. 6 d. by 2 s. 6 d. a Shilling the Integer.

Six Pence being reduced to the Decimal of a Shilling, the Work will ſtand thus:

2,5
2,5
———
125
50
———
6,25 = 6 s. 3 d.

Multiply 4 l. 12 s. 6 d. by 5 l. 15 s. 6 d.
Decimal anſwering 12 s. 6 d. is ,625.
Decimal anſwering 15. 6. is ,775.
Then multiply 4,625
By 5,775
———
23125
32375
32375
23125
———
26,709375 = 26 l. 14 s. 2 d. $\frac{1}{4}$.

Here follows a Method to contract your Product to what Number of Decimals you pleaſe, as in the laſt Example. I would have the Product conſiſt but of three Places of Decimals. See the Work.

Multiply 5,775 by 4,625, and to have but three Places of Decimals.

5,775 Multiplicand.
526,4 Multiplier inverted.

23101 Mind, one is brought in for the in-
3465 {creaſe of the Figures.
115
28

26,709

N. B. That you place Unity of your Multiplier under as many Places diſtant from Unity in your Multiplicand, as you would have Decimals in your Product; then multiply as uſual, only mind that you begin your Multiplicand with that Figure that ſtands over the Figure you multiply by, having reſpect to the Increaſe that would come from the following Figures of the Multiplicand, placing every ſingle Product exactly at the right Hand, contrary to the common Way, then add them, and cut off as many as you deſigned, as you ſee by the preceding Example. But the common Way is the beſt.

Diviſion of Decimals

DIffers nothing from Diviſion of Integers, either in placing the Numbers, or in the Work itſelf, the Difficulty being in diſcovering the true Value of the Quotient, which to do, obſerve the following Rule.

Rule. The Dividend muſt contain as many Decimals as the Diviſor, and the Quotient both together; that is, if there be 7 Decimals in the Dividend, and 3 in the Diviſor, there muſt be 4 in the Quotient, or if there be 7 in the Dividend, and none in the Diviſor, there muſt be 7 in the Quotient, always minding to keep the Dividend equal in Number of Decimals to the Diviſor and Quotient.

EXAMPLE.

Divide 1342,332 by 36

```
36)1342,332(37,287
   108
   ---
    262
    252
    ---
     103
      72
     ---
      313
      288
      ---
       252
       252
       ---
         0
```

In the above Example, the Dividend is a mixt Number, and contains 3 Decimals, but the Diviſor is an Integer; then by the Rule, there muſt be 3 Decimals in the Quotient.

EXAMPLE 2d.

Divide 17,3592 by 4,6

```
4,6)17,2592(3,752
    138
    ---
     345
     322
     ---
      239
      230
      ---
        92
        92
        --
         0
```

In this ſecond Example, both the Dividend and Diviſor are mixt Numbers, and there is 4 Decimals in the Dividend, and 1 Decimal in the Diviſor; then there muſt be 3 Decimals in the Quotient, to make the Numbers in Diviſor and Quotient equal to the Number of Decimals in the Dividend.

EXAMPLE 3d.

Divide 4672,565 by 25,635

```
25,635)4672,565(182
       2563 5
       ------
       2109 06
       2050 80
       -------
         58 265
         51 270
         ------
          6 995 Remainder.
```

In this third Example, there is a like Number of Decimals in both the Dividend and Diviſor, ſo the Quotient will be an Integer; and if you add Cyphers to the Remainder, and divide as before, you may bring the Quotient to what Number of Decimals you pleaſe.

EXAMPLE 4th.

Divide ,75 by ,0125

0125),7500(60

750

———

00

In this 4th Example, both the Dividend and the Diviſor are Decimals; and becauſe I cannot divide the Dividend by the Diviſor, I add Cyphers to the Dividend, to make the Number of decimal Places in the Dividend equal to thoſe in the Diviſor, and the Quotient is an Integer. And you may obſerve, that Multiplication of Fractions decreaſeth their Value, ſo Diviſion of Fractions increaſeth their Value contrary to the Nature of Integers. This Example is the ſame as if it were demanded to divide 15 s. by 3 d. a Pound being the Integer; For the Decimal of 15 s. is ,75, and that of 3 d. is ,0125, as you may find by Reduction; and the Quotient will be 60

EXAMPLE 5th.

Divide 1425 by ,6252

,6252)1425,0000(2279
12504

17460
12504

49560
43764

57960
56268

1692 Remainder.

You ſee that before the Diviſion can be made, you muſt add a competent Number of Cyphers to the Dividend, as I have done, to the Number of 4, which is as many as I have Decimals in my Diviſor, and then the Quotient will be Integers; but if you would have Decimals in the Quotient, then you muſt add as many Decimals to the Dividend, more than there is in the Diviſor, as you would have decimal Places in your Quotient; or if you add Cyphers to the Remainder in this Example, and proceed on with your Diviſion, all the Numbers will be Decimals in the Quotient after. As if you would add 7 Decimals to the Dividend at firſt, or 3 to the Remainder, (for Cyphers are the ſame as Decimals) the Quotient you will find to be 2279,270, and the Remainder 3960.

EXAMPLE 6th.

Divide, ,358845 by 47

```
47),358845(,007635
    329
    ---
     298
     282
     ---
      164
      141
      ---
       235
       235
       ---
         0
```

In this Example, the Dividend is a Decimal, and the Diviſor an Integer, and when the Diviſion is ended, there is but 4 Decimals in the Quotient, and there is 6 in the Dividend, for which Reaſon, I add 2 Cyphers to the left Hand of my Quotient, to make them equal; as there were none in the Diviſor, according to the Rule.

EXAMPLE 7th.

Divide 184,41475 by 32,5

```
32,5)184,41475(5,6743
     1625
     ------
      2191
      1950
      ------
       2414
       2275
       ------
        1397
        1300
        ------
         975
         975
         ------
           0
```

In this Example, there is five Decimals in the Dividend, and but one in the Diviſor, then there muſt be 4 in the Quotient.

If you have any decimal Fraction to divide by a Unit, with any Number of Cyphers annex'd, it is but removing the Comma or Mark to diſtinguiſh the Decimal, as many Places to the left Hand, as you have Cyphers annexed to the Unit.

EXAMPLES.

Divide 14,4 by 10, the Quotient is 1,44.
Divide 1,44 by 10, the Quotient is ,144.
Divide 1,44 by 100, the Quotient is ,0144.
Divide 1,44 by 1000, the Quotient is ,00144.

N. B. If the Dividend be greater than the Diviſor, the Quotient will be either an Integer or a mixt Number; but if the Dividend be leſs than the Diviſor, the Quotient will be a Decimal.

Multiplication and Diviſion in Decimals (as in Integers) interchangeably prove each other.

To prove Multiplication, divide the Product by the Multiplier, gives in the Quotient the Multiplicand; or dividing the Product by the Multiplicand, gives in the Quotient the Multiplier.

To prove Diviſion, multiply the Quotient by the Diviſor, gives the Dividend.

What Diviſor is that, by which dividing 746 ſhall give a Quotient equal to the Product of the ſame Number multiplied by 4? Facit ,25

Found by dividing an Unit with Cyphers by the Multiplicator thus.

```
4)1,00(,25
   8
  ---
   20
   20
  ---
    0
```

The Proof.

```
 749          ,25)749,00(2996
   4              50
-----            ----
2996              249
                  225
                 ----
                   240
                   225
                  ----
                    150
                    150
                   ----
                      0
```

In this the Product and Quotient are the ſame, and ſo of any other.

What Multiplicator is that by which multiplying 749, ſhall give a Product equal to the Quotient of the ſame Number divided by ,25? Facit 4.

Found by dividing Unity with Cyphers annexed by the Diviſor, thus:

```
,25)1,00(4
    1 00
    ----
       0
```

If the Dividend be 6125, and the Quotient 49000, what is the Diviſor? Facit ,125 found by dividing the Dividend with Cyphers, added by the Quotient.

If what has been ſaid in Decimal Fractions be underſtood, any Queſtion may be anſwered in any of the Rules of Three in Decimals, by obſerving the Rules given in Reduction, &c.

Single Fellowſhip

IS when two or more Men trade together with one common Stock, and when they have gained or loſt, it ſhews how much is each Man's particular Share of the Gain or Loſs, without any regard to Time. The Rule is this.

Rule. As the whole Stock is to the whole Gain or Loſs, ſo is each Man's particular Stock to his particular Gain or Loſs.

EXAMPLE.

Four Merchants, *A*, *B*, *C*, *D*, trading together, made a common Stock of Money, to the Value of 240l. whereof *A.* put in 30l. *B.* 50l. *C.* 60l. and *D.* 100l. This Stock they occupied till they gained 2760l. what did each Man gain.

All Queſtions in this Rule are anſwered by ſo many Statings in the Rule of Three, as there are Partners. See the Work of this Queſtion.

If 240 : 2760 :: 30 : 345 = A's Gain.
If 240 : 2760 :: 50 : 575 = B's Gain.
If 240 : 2760 :: 60 : 690 = C's Gain.
If 240 : 2760 :: 100 : 1150 = D's Gain.

The Anſwer is obtained by × the ſecond and third Term together, and dividing that Product by the firſt. And the Proof is adding up the ſeveral Gains into one Sum, which gives the total Gain = 2760.

E X A M P L E 2d.

If a Bankrupt's Goods be valued at 500l. and he be 1000l. in Debt to 4 Men, *viz. A*, *B*, *C*, *D*, of which 160l. is due to *A*, 200l. to *B*, 300l. to *C*, 340l. to *D*, now, what ought to be each Man's Share?

If 1000 : 500 :: 160 : 80 = A's Share.
If 1000 : 500 :: 200 : 100 = B's Share.
If 1000 : 500 :: 300 : 150 = C's Share.
If 1000 : 500 :: 340 : 170 = D's Share.

And ſo of any other Queſtion in this Rule.

* * *

Fellowſhip with Time

IS wrought with this Rule. Multiply each Man's particular Stock by his Time; then ſay, as the Sum of theſe Products is to whole Gain or Loſs, ſo is each Man's particular Product to his particular Gain or Loſs.

E X A M P L E.

Four Merchants *A*, *B*, *C*, *D*, made a common Stock, which at the Year's end was increaſed to 35,145l. of this Stock *A* put in 669l. which he took out at 10 Month's end; *B* laid in 810l. for 8 Months; *C* laid in 900l. for 7 Months; and *D* put in 1040l. for 12 Months. Now what is each Man's Share of the Stock?

669 × 10 = 6690 A's Stock, into his Time.
810 × 8 = 6480 B's Stock, into his Time.
900 × 7 = 6300 C's Stock, into his Time.
1940 × 12 = 12480 D's Stock, into his Time.

31950

If 31950 : 35145 :: 6690 : 7359 = A's.
If 31950 : 35145 :: 6480 : 7128 = B's.
If 31950 : 35145 :: 6300 : 6930 = C's.
If 31950 : 35145 :: 12480 : 13728 = D's.

Proof 35145

For if you add each Man's Share of the Stock into one Sum, it will produce the whole Stock, if your Work be true.

EXAMPLE.

There is in a Cathedral Church, 20 Canons, and 30 Vicars, and they ſpend in a Year 2600 l. but every Canon muſt have five times as much as a Vicar, how much is each Man's yearly Income?

In anſwering ſuch Queſtions as this, you muſt multiply the Number of Perſons by their difference of Portions, and add the Products into one Sum, as you did by Time.

So that 20 × 5 = 100, to which add 30 = 130, then
If 130 : 2600 :: 100 : 2000 l. for the 20 Canons.
If 130 : 2600 :: 30 : 600 l. for the 30 Vicars.

And if you divide 2000 by 20, it gives 100 l. for each Canon; and 600 divided by 30, gives 20 l. for each Vicar.

The Rule of Alligation

TEacheth, how to mix or unite many Simples or Particulars, into one Sum or Maſs, according to any Price or Proportion required. I ſhall divide it into four Varieties, that when a Queſtion is propounded, you may ſee what Variety it falls under.

EXAMPLE.

If a Corn Factor mix 5 Buſhels of Wheat at 6 Shillings the Buſhel, with 12 Buſhels of Rye at 4 Shilling the Buſhel, with 8 Buſhels of Beans at 5 Shillings the Buſhel, and with 18 Buſhels of Barley at 2 Shillings and 6 Pence the Buſhel ; the Price of one Buſhel of this Mixture is demanded ?

5	Buſhels at 6s. the Buſhel, is	1 : 10 : 0
12	Buſhels at 4s. the Buſhel, is	2 : 08 : 0
8	Buſhels at 5s. the Buſhel, is	2 : 00 : 0
18	Buſhels at 2s. 6d. the Buſh. is	2 : 05 : 0
43	Buſhels. The total Value is	8 : 03 : 0

B. B.

Then ſay, if 43 : 8l. 3^{s}. : : 1 ; or if you bring 3 Shillings into the Decimal of a Pound, it will ſtand thus :

B : L. B. L.

If 43 : 8,15 :: 1 : ,1895 = to 3s. 9d. ½

VARIETY 2d.

In this the Price of the Simples is expreſſed, but no Quantity given, and it is demanded how much of each Simple we muſt take to ſell one Quantity or Meaſure at a Price propounded ; the whole of this Variety is in linking the Extreams truly together, and taking the Differences betwixt them and the Mean ; and theſe Differences are the true Quantities ſought. But inſtead of linking them, if you make the ſame Mark to any two you would join together, it will be full as plain and more quickly done.

Examples will make it eaſy.

Suppoſe a Man hath Spices, ſome at 9d. the Pound, ſome at 12d. ſome at 24d. and ſome at 30d. how much of each Sort muſt he take that he ſell a Pound for 20d. ? Set down the Prices and Mean in the following Manner.

Mean 20	9×	10
	12✓	4
	24✓	8
	30×	11

Here you ſee that 9 and 30 are markt alike, and ſo is 12 and 24. And the Difference betwixt 9 and 20 is 11, which place to its like Mark 30, and the Difference betwixt 20 and 30 is 10, which place to 9, and the Difference betwixt 12 and 20 is 8, which place to 24 ; and the Difference betwixt 24 and 20 is 4, which place to 12 ; and the Differences thus found is the Anſwer to the Queſtion. For as often as he takes 10 Pound, of 9d. a Pound, he muſt take 4 of 12d. the Pound, and 8 of 24d. the Pound, and 11 of 30d. the Pound, as you may prove by the laſt Queſtion. You may have many Anſwers to ſuch Queſtions as this, iſ you mark one leſs than the mean Price with one greater. As in the Queſtion before now, 9 and 24 is markt alike, ſo is the Prices 12 and 30. And the Difference betwixt 9 and 20 is 11, which place to 24 ; and the Difference betwixt

20	9×	4
	12✓	10
	24×	11
	30✓	8

twixt 24 and 20 is 4, which place to 9; and the Difference betwixt 12 and 20 is 8, which place to 30; and the Difference betwixt 30 and 20 is 10, which place to 12. And thoſe Differences are the true Anſwer to the Queſtion as before. And you may have other Anſwers if you vary the marking of the Prices by the above Direction.

N. B. If you add the Differences of thoſe two Anſwers together, the ſeveral Sums will be the Anſwer; or if you multiply the Differences by one common Multiplier, the ſeveral Products will be the Anſwer, as you may try at your Pleaſure.

A Merchant hath Wines; Canary at 24 d. the Quart, Sherry at 16 d. the Quart, and Malaga at 12 d. the Quart; how much of each Sort muſt he take to ſell a Quart for 18 d. ?

18	24✓	2.6	8
	16✓		6
	12✓		6

This Queſtion you ſee admits but of one Way of marking, becauſe there is but one Quantity larger than the mean Price, which is markt to both the other Quantities, which are under the mean Price. So that he muſt have of the Canary 8 Quarts, and 6 Quarts of each of the other, or in that Proportion.

VARIETY 3d.

In this we have the the Price of all the Simples, and the Quantity of one given, to find the Quantity of all the reſt, ſo as one Meaſure or Quantity may bear a mean Rate or Price propounded. Which to do, obſerve the following Rule.

As the Difference ſtanding againſt the given Quantity, is to the reſt of any Differences beſides :: So is the given Quantity, to the Quantities ſought. Each to his reſpective Difference.

EXAMPLE.

A Tobacconiſt hath 30℔. of Tobacco, at 24 d. the Pound, which he would mix with ſome of 12 d. ſome at 9 d. and ſome at 7 d. and he would know how much of the ſaid leſs Prices he muſt mix with the 30 ℔. of the beſt, that he may ſell a Pound for 16d.? Set them down as in the laſt Example.

16	24✓	4. 7. 9.	20
	12✓	8	8
	9✓	8	8
	7✓	8	8

N. B. That 24 is markt to all the other, ſo you muſt take the Differences betwixt every Price and the Mean, and place it to 24, and the Sum of theſe is 20. Then you muſt take the Difference betwixt 24 and the mean Price which is 8, and Place it againſt every Price, becauſe 24 is markt to every Price; then ſay

If 20 : 8 :: 30 : 12

So that he muſt have 12 ℔. of each Quantity to mix with the 30 ℔. of the beſt, that a Pound may be ſold for 16 d.

EXAMPLE 2d.

A Goldſmith hath 20 Ounces of Gold, at 20 Carracts fine, and would mix it with ſome at 22 Carracts fine, and ſome at 24 Carracts fine; how much

of

22 and 24 Carracts fine, and how much Alloy muſt he mix with the 20 Ounces of 20 Carracts fine, to have an Ounce, and conſequently the whole Maſs at 18 Carracts fine.

Note, That Alloy is a Sort of courſe Silver, or Copper, or ſome baſe Metal, with which Goldſmiths mix Gold or Silver, to abate the Fineneſs thereof.

An Ounce of Gold is divided into 24 Parts called Carracts, and an Ounce of Silver into 20 Parts, called Penny Weights; therefore to diſtinguiſh Fineneſs of Metals, ſuch Gold as will abide the Fire without Loſs, is accounted 24 Carracts fine; if it loſe 2 Carracts in Trial, it is then called 22 Carracts Fine, &c.

Silver is valued in Ounces, and a Pound of Silver which loſeth nothing in Trial, is called 12 Ounces fine; but if it loſeth 2 Penny Weight, it is then ſaid to be 11 Ounces, 18 Penny Weight fine. Set down the Values in Order, as uſual, with the mean Value, and in the Place of the Alloy, (becauſe it is not accounted of any Value,) place a Cypher, then take the Differences, which by marking you may ſee, will be the ſame, except only in the Place of Alloy.

18	20✓	18	18
	22✓	18	18
	24✓	18	18
	0✓	2.4.6	12

Then ſay If 18 : 18 : : 20 : 20

If 18 : 18 : : 20 : 20

If 18 : 12 : : 20 : $13\frac{1}{3}$

So you ſee, that with the 20 Ounces of 20 Carracts fine, you muſt mix 20 of 22 Carracts fine, and 20 of 24 Carracts fine, and 13 Ounces and $\frac{1}{3}$ of Alloy, that ſo an Ounce will bear 18 Carracts fine. You may obſerve, that every

Quantity is markt with the ſame Mark as the Alloy; ſo that the Difference betwixt 20 and 18 is 2, betwixt 22 and 18 is 4, and betwixt 24 and 18 is 6, whoſe Sum is 12, then the Difference betwixt the Alloy and 18 is 18, which place againſt every Quantity, as it has the ſame Mark.

VARIETY 4th.

In this the Prices of every Simple is expreſſed, and the mean Rate or Price; and it is required to find how much of each Sort muſt be taken to make up a certain Quantity propounded, agreeable to the mean Rate given? Which to do, obſerve the Rule following.

As the total Sum of the Differences is to the total Quantity given, ſo is any particular Difference to its particular Quantity ſought.

EXAMPLE.

A Merchant would mix 4 Sorts of Wine, of ſeveral Prices, one at 6 Shillings *per* Gallon, another at 8 Shillings, the third at 11 Shillings, and the fourth at 15 Shillings *per* Gallon; of theſe Wines he would have a Mixture made to contain 50 Gallons, and the Price of one Gallon to be 9 Shillings. How much muſt he take of every Sort of Wine?

$$9\left|\begin{array}{l} 6\checkmark \\ 8\times \\ 11\times \\ 15\checkmark \end{array}\middle|\begin{array}{r} 6 \\ 2 \\ 1 \\ 3 \end{array}\right|$$

$$\overline{12} = \text{Sum of the Differences; then ſay,}$$

If 12 : 50 :: 6 : 25=the Gal. at 6s *per* Gallon
If 12 : 50 :: 2 : $8\frac{1}{3}$=the Gal. at 8s *per* Gal.
If 12 : 50 :: 1 : $4\frac{1}{6}$=the Gal. at 11s *per* Gal.
If 12 : 50 :: 3 : $12\frac{1}{2}$=the Gal. at 15s *per* Gal.
——
50 Proof.

For the $\frac{1}{3}$ and $\frac{1}{6}$ of any Thing is = to $\frac{1}{2}$. You may have other Anſwers, if you vary the marking of the Quantities by the Rule given in the ſecond Variety. And the Proof of the Money is found by the firſt Variety.

Extraction of the Square Root.

THE Square Root of a Number, is a Number which being multiplied by itſelf, produceth the given Number; or, the Square Root of a Number may be defined to be a mean Proportional between the given Number and Unity. Here follows a Table of Squares as far as the firſt 9 Digits.

Root	1	2	3	4	5	6	7	8	9
Square	1	4	9	16	25	36	49	64	81

And when it is required to extract the Square Root of any given Number, we have nothing to do but to find that equal Number of which it was compoſed; ſo if the Root of 25 were required, it would be found 5 for 5 × 5 = 25, as you may ſee by the Table. Here 5 is the Root or firſt Power, and the Number

25 is called the Square, or ſecond Power. Of Numbers to be extracted, are three Sorts. Firſt Single, as in the Table, which are compoſed of the 9 Digits.

Secondly, Compound, that are compoſed of more Figures than one, as of 16, whoſe Square is 256, for 16×16=256, or 48×48=2304, or 100×100=10000. Here the Roots are 16.48 and 100, and the Squares 256.2304 and 10000. Third, Irrational are all ſuch Squares, whoſe Roots cannot be diſcovered by Art exactly, neither in whole Numbers or Fractions; but ſomething will ſtill remain, there being no Proportion found betwixt an irrational Number and its Root, ſuch Numbers are 5. 19. 21. 75. 156. 867. &c. The Extraction of the Square Root is not much unlike Diviſion, only there our Diviſor is fixed, here we are to find a new one for each Opperation. The Root of any ſingle Square Number is found by Inſpection, as may be ſeen by the foregoing Table. But if it be a compound Square Number, it muſt be prepared by pointing thus: make a Point under the Units Place, and omitting one, point every other Figure; and as many Points as your Number contains, ſo many Figures will your Root conſiſt of.

Examples will make it eaſy; but if the Learner firſt take a Number, and ſquare it, and extract the Root of the Product, it will facilitate the Manner uſed in the Extraction.

EXAMPLE 1ſt.

If 246×246=60516, then find the Root of 60516, which being pointed as directed, it will ſtand thus:

$\dot{6}0\dot{5}1\dot{6}(246=$ the Root
4

Root × 2 = 44)205
176

Root × 2 = 486)2916
2916

0

You know at the Beginning of the Work what the Root will be, then point it, beginning at the Units Place, and miſs every other Figure, and the Points will ſtand under the Numbers 6. 5. and 6. which ſhews you that there will be three Figures in the Root = to the Number of Points. Then take the neareſt Square by your Table, which is the ſame as your laſt Point, or under it in Value, as you will find 2, which place in the Root, and ſet its Square under the 6, and ſubtract it, and the Remainder is 2; then bring down your next Period 05, and put it to the Remainder, and it will be 205; then draw a Line as in Diviſion, and × your Root by 2, and it makes 4, which place as your Diviſor, leaving Room towards the right Hand, and place your next Figure, that will be 4, in your Root in that Place; then multiply as in Diviſion, and place the Product, which is 176 under 205, and the Remainder will be 29, to which bring down your next Period 16, and join it to the Remainder, and it will make 2916; then draw a Line as before, to place your Diviſor in, and double your Root as before, and it will be 48, ſtill leaving Room towards the right Hand to place the next Figure in your Root there, which is 6, and the Diviſor is then 486, which being × by 6, the laſt Figure in the Root, the Product is 2916, and there is no Remainder; which ſhews that your Work is right. Obſerve this in all the following Examples.

EXAMPLE 2d.

176×176=30976, and its Root is required to be found by the Method uſed in extracting the Square Root; it being pointed, the Work will ſtand in this Manner:

30976)176 Root
1
—
27)209
189
—
346)2076
2076
—
0

EXAMPLE 3d.

674×674=454276

What is the Square Root of 454276? See the Work.

```
      454276(674=Root.
      36
     ------
  127)942
      889
     -------
 1344)5376
      5376
      -----
         0
```

EXAMPLE 4th. with Decimals

87,25×87,25=7612,5625

What is the Square Root of 7612,5625? See the Work.

```
        7612,5625)87,25=Root.
        64
        ----
   167)1212
       1169
       -------
  1742)4356
       3484
       -----
 17445)87225
       87225
       -----
           0
```

N. B. If there had been an odd Number of Decimals, you muſt have put a Cypher towards the right Hand, to make them even, and then begun to point as directed; or if you begin at the right Hand of the Decimals, it would have been the ſame, and the Points ſhew you the Number of whole Numbers and Decimals that will be in the Root. For thoſe Points under the Decimals, ſhew the Number of Decimals in the Root, and thoſe under the whole Numbers, the whole Numbers.

EXAMPLE 5th.

17,75×17,75=315,0625

What is the Square Root of $\underset{\cdot}{3}1\underset{\cdot}{5},0\underset{\cdot}{6}2\underset{\cdot}{5}$?

$$\begin{array}{rl}
 & \underset{\cdot}{3}1\underset{\cdot}{5},0\underset{\cdot}{6}2\underset{\cdot}{5}(17,75=\text{Root.}\\
 & 1\\
\hline
27) & 215\\
 & 189\\
\hline
347) & 2606\\
 & 2429\\
\hline
3545) & 17725\\
 & 17725\\
\hline
 & 0
\end{array}$$

If the Learner obſerve the Directions given, by this Time the Method of extracting the Square Root will be familiar to him; and I ſhall now proceed to extract the Root of a Number, without knowing from what Number it grew.

What is the Root of 451584(672=Root?
36

127)915
889

1342)2684
2684

0

N. B. If you multiply the Root by itſelf, the Product will be the Number given to be extracted; and if there be a Remainder, you muſt add it to the Product, and the Sum will be the given Number, if your Work is right.

Extract the Square Root of 4624(68=Root.
36

128)1024
1024

0

```
What is the Root of 784762(885
                    64
                    ——
              168)1447
                  1344
                  ————
             1765)10362
                   8825
                   ————
                   1537 Remainder.
```

Now $885\times885=783225+1537$ the Remainder $=784762$; which proves the Work right.

N. B. If you add two Cyphers to the Remainder, you might have found the firſt Place of Decimals, by proceeding in every Reſpect as before; and then two more Cyphers being added to the next Remainder, and proceeding as before, you would have had the ſecond Place in Decimals; and by adding two Cyphers to every Remainder, you might bring the Root to as many Places of Decimals as you pleaſe.

But if you would have your Root expreſſed as a Vulgar Fraction, do thus: After you have found the Integral Part of your Root, to its Quadruple, add Unity for the Denominator of the Fractional Part, and the Remainder doubled is the Numerator; ſo the Root, by this Method will be $885\frac{3074}{3541}$ nearly.

Extract the Root of 674637

```
      674637(821,36289
      64
      ---
    162)346
        324
        ----
   1641)2237
         1641
         -----
  16423)59600
          49269
          ------
 164266)1033100
          985596
          --------
 1642722)4750400
          3285444
          ---------
 16427248)146495600
           131417984
           -----------
 164272569)1507761600
           1478453121
           ------------
             29308479
```

In this Example, the Root is extracted to five Places of Decimals, which is near enough: for if Unity was divided into a Thouſand Parts, it would not want three of them, and if it was carried two Decimals more, it would not want 2 Parts if Unity were divided into a Hundred Thouſand Parts; as you may prove at your Pleaſure.

Queſtions to Exerciſe the Square Root.

TO find a mean Proportional between any two Numbers.

R U L E.

The Square Root of the Product of the given Numbers, is the mean Proportional ſought.

E X A M P L E. 1ſt.

Find a mean Proportional betwixt 49 and 64.
Then 49 × 64 = 3136.
And 3136(56 = The Root and mean ſought.
25 For as 49 : 56 :: 56 : 64.

106)636
636

0

To find the Side of a Square = in Area to any given Superficies whatſoever.

R U L E.

Find the Square Root of the Area, and that Root is the Side of the Square ſought.

If the Content of a Circle be 160, what is the Side of a Square = in Area to the given Circle ?

```
160(12,64911 =the Side of a Square,
1             equal in Area to the
———           given Circle.
22)60
   44
   ————
246)16,00
    14 76
    ——————
2524)12 400
     10 096
     ———————
25289)23 0400
       22 7601
       ————————
252981)279900
       252981
       ————————
2529821)2691900
        2529821
        ———————
         162079 Remainder.
```

N. B. After you have brought out the Integers 12, and would have the Root expreſſed as a Vulgar Fraction, by the Rule given before, you will find it to be $12\frac{32}{49}$, but the Decimals is the moſt exact. And 12,64911 is the Side of a Square, whoſe Area is 160 Square Perches, or one Acre.

Any Number of Soldiers being propoſed, to order them into a Square Battalia; that is, there ſhall be as many in Rank as in File. *Rule.* Extract the Square Root of the given Number of Men, and that Root is the Anſwer.

Suppoſe a General had an Army of 138384 Men, and he would form them into a Square Battalia; how many Men muſt be in Rank and File?

```
 138384(372=the No. in Rank and File.
  9
 ----
67)483
   469
   ----
742)1484
    1484
    ----
       0
```

Any Number of Men being propoſed to order them into a double (triple, or Quadruple, &c.) Battalia, that is, having twice (or three times, or four times, &c.) as many in Rank as in File.

R U L E.

Extract the Square Root of ½ (or ⅓ or ¼, &c. the given Number of Men, and that is the Number of Men in File, and double (or triple, or Quadruple, &c.) that Number, is the Number in Rank.

Let 8192 Men be formed into an Oblong Battalia, ſo that the Number of Men in Rank may be double the Number of Men in File? And 8192 ÷ 2 = 4096. Then 4096(64=the Number of Men in File and 64 × 2 = 128 the No. in Rank.

```
 4096(64
 36
 ----
124)496
    496
    ---
      0
```

Any Number of Men being propoſed to be put in Battalia, and a certain Number aſſigned to be put in Rank, to find the Number in File; or a certain Number aſſigned to be put in File, to find the Number in Rank.

R U L E.

As the Number of Men aſſigned to be put in Rank, is to the whole Number of Men; ſo is Unity to the Number in File. And as the Number of Men aſſigned to be put in File is to the whole Number of Men; ſo is Unity to the Number in Rank.

Let 12132 Men be placed in Battalia, ſo that there ſhall be 18 in Rank; how many muſt be in File?

As 18 : 12132 :: 1 : 674 to the Number in File.

Any Number of Men being given to be put in Battalia, and their Number and Diſtance in Rank and File being given to find how much Ground they will occupy. As 1 is to the Diſtance in Rank; ſo is the Number in Rank leſs 1 to a 4th. And as 1 is to the Diſtance in File, ſo is the Number in File leſs 1 to a 4th. Then the Rectangle or Product of theſe 4ths, gives the Quantity of Ground ſought, in the ſame Meaſure as the Diſtance was given.

E X A M P L E.

Let it be required to place 6068 Men in Battalia, ſo that there be 82 in Rank, and 74 in File, and they all to ſtand in Order, that is, three Feet diſtant in Rank, and as much in File; how much Ground will they occupy? For

$$\left.\begin{array}{l} \text{As } 1 : 3 :: 81 : 243 \\ \text{As } 1 : 3 :: 73 : 219 \end{array}\right\} \text{And } 243 \times 219 = 53217. \quad \text{And,}$$

So many Square Feet of Ground they will occupy.

Though theſe two laſt Queſtions do not belong to the Square Root, I thought it would not be diſagreeable to give them a Place here, as they relate to Military Affairs.

Any Number of Men being propoſed, to place them in Battalia, ſo that the Number of Men in Rank, ſhall be to the Number of Men in File, according to any Proportion aſſigned; or the Number in File to the Number in Rank.

RULE.

As the Ratio reſpecting the Rank, is to the Ratio reſpecting the File; ſo is the whole Number of Men to a 4th, whoſe Square Root is the Number of Men in File, &c.

Let 864 Men be placed in Battalia, ſo the Number of Men in Rank may be to the Number in File as 3 to 2; how many muſt there be in Rank, and how many in File?

Then as 3 : 2 :: 864 : 576, whoſe Root is 24 Number in File.

And as 2 : 3 :: 864 : 1296, whoſe Root is 36 Number in Rank.

The Area of a Circle being given, to find the Diameter.

RULE.

AS 355 is to 452, or as 1 to 1,273239, ſo is the Area to the Square of the Diameter.

What Length of Cord will ſerve to tye to a Cow-Tail, the other End fixed to the Ground, to let her have Liberty of eating an Acre of Graſs and no more, ſuppoſing the Cow and Tail to be one Perch, or five Yards and a Half?

Say, as 355 : 452 :: 160 being the Area of a Circle, whoſe Content is an Acre, to 203,7183. whoſe Square Root is 14,2732, the Diameter ſought, whoſe Radius is 7,1366, from which ſubtract one Peach for the Cow and Tail; reſt 6,1366 for the Length of the Cord ſought.

The Area of a Circle given to find the Periphery.

RULE.

Say as 113 : 1420 :: or, as 1 to 12,56637, ſo is the Area to the Square of the Periphery. So if the Area of a Circle be 160, the Periphery will be found 44.84.

A certain Number of Men ſpent at a Reckoning the Sum of 14l. 8s. and every Man paid as many Sixpences as there were Men in Company. How many Men were there?

R U L E.

Bring the Money into Sixpences, and extract the Root of that Product, which Root will be the Number of Men. See the Work.

```
 l. : s.
 14 : 8
  20
 ------
 288
   2
 ------
 576 (24=the Number of Men.
 4
 ------
44)176
   176
 ------
     0
```

In the 47th Propoſition of the firſt Book of Euclid, it is proved that the Square of the Hypothenuſe, or longeſt Side of a right-angled Triangle, is equal to the Sum of the Squares of the Baſe or Perpendicular; or the other two Sides.

Let the Baſe of a right angled Triangle be 36 Feet, and the Perpendicular Height be 48 Feet, what Length will the Hypothenuſe, or longeſt Side of the Triangle be?

$36 \times 36 = 1296$. And $48 \times 48 = 2304$.

Then $1296+2304=3600$, whoſe Square Root is 60, the Length of the longeſt Side in Feet ſought.

The Hypothenuſe, or longeſt Side of a right angled Triangle be 60 Feet, and the Baſe 48 Feet, what is the Perpendicular?

60 × 60 = 3600. And 48 × 48 = 2304. Then 3600 — 2304 = 1296, whoſe Root is 36 the Length of the Side ſought.

Let the Hypothenuſe of a right angled Triangle be 60 Feet, and the perpendicular Height 36 Feet, what is the Baſe Line?

60 × 60 = 3600. And 36 × 36 = 1296. Then 3600 — 1296 = 2304, whoſe Root is 48 Feet, the length of the Baſe Line.

Thoſe three Queſtions mutually prove one another. And this much ſhall ſuffice for the Square Root, and I ſhall proceed to the Cube.

Extraction of the Cube Root.

THE Cube Root of a Number, is a Number, which being multiplied by itſelf twice, produceth the given Number. Thus the Cube Root, of 125 is 5: For 5 × 5 × 5 = 125. And the Cube Root of 512 will be found 8. And ſo of other Numbers, as in the following Table.

The TABLE.

Root.	Cube.
1	1
2	8
3	27
4	64
5	125
6	216
7	343
8	512
9	729

The Root of any ſingle Cube Number is found by Inſpection, as in the Table.

But if it be a compound Cube Number, it muſt be pointed thus: make a Point under the Units Place, and omitting two, point every third Figure, and as many Points as your Number contains, ſo many Figures will your Root conſiſt of. Then proceed by the following Direction.

Take the neareſt Cube Number to your firſt Period to the left Hand, by the Table, whoſe Root place in the Quotient, then ſubtract its Cube from the Period, and to the Remainder (if any) bring down three Figures, or your next Period, which call your Dividend, then call the Quotient Figure, with a Cypher joined to it, *r*; and ſquare it, and × that Square by 3, and the Product you may call your Diviſor. And then proceed as in Diviſion, only with this Difference, call your next Quotient

Figure

Figure *e*, and × your Diviſor by it, and place it under your Dividend, then × your *r* by 3, and that Product × by the Square of that Figure you call'd *e*; laſtly cube the Figure you call'd *e*, and place thoſe two laſt under the Dividend alſo, and add the three ſeveral Sums into one, and ſubtract this laſt from your Dividend, and to the Remainder bring down your next Period, and proceed as before. Examples will make it more intelligible than a multitude of Words.

Cube 68, that is 68×68×68=314432.

Then extract the Cube Root of 314432. Point it as directed, and it will ſtand thus $31\dot{4}43\dot{2}$ (68

216

60=*r*, then 3*rr* that is ——

60×60×3=10800)98432=Divid.

86400=3*rre*

11520=3*ree*

512=*eee*

98432

0

N. B. That 8 the ſecond Figure in your Quotient is called *e*; and if your Root conſiſt of more Figures than two, then the *r* changes its Name every Period you bring down, as you may may ſee in the following Example.

EXAMPLE 2d.

$672 \times 672 \times 672 = 303464448$

What is the Cube Root of 303464448? See the Work.

$30\dot{3}46\dot{4}44\dot{8}(672$
216

$3rr = 10800)87464 =$ the Dividend.

$75600 = 3rre$
$8820 = 3ree$
$343 = eee$

$84763 =$ The Subducend.

$3rr = 1346700)2701448 =$ The Dividend.

$2693400 = 3rre$
$8040 = 3ree$
$8 = eee$

$2701448 =$ The Subducend.

0

When you bring down your ſecond and laſt Period, the *r* becomes 670, and your *e* 2, and ſo proceed to change your *r* every Period you bring down.

Find the Cube Root of $30\dot{7}27\dot{3}58\dot{0}99\dot{2}$? See the Work.

$30\dot{7}27\dot{3}58\dot{0}99\dot{2}(6748=$Root.
216

$3rr=10800)91273=$Dividend.

$75600=3rre$, here $e=7$:
$8820=3ree$
$343=eee$

$84763=$Subducend.

$3rr=1346700)6510580=$Dividend the 2d.

$5386800=3rre$, here $e=4$.
$32160=3ree$
$64=eee$

$5419024=$Subducend.

$3rr=136282800)1091556992=$Dividend 3d.

$1090262400=3rre$here $e=8$
$1294080=3ree$
$512=eee$

$1091556992=$Subducend.

Reſt—00

In this Example your firſt r was$=60$, your firſt $e=7$, your ſecond $r=670$, your ſecond $e=4$, and your laſt r=6740, and your laſt e 8; and in this Manner you muſt go on till your Work is finiſhed, and if there had been a Remainder, you muſt add three Cyphers to it for a Decimal, &c.

Extract the Cube Root of 282; or find the Side of a cubical Veſſel, which ſhall contain a Gallon of Ale, being 282 ſolid Inches?

282(6,557=Root.
216

$3rr$=10800)66000 Dividend.

54000=$3rre$
4500=$3ree$
125=eee

58625=Subducend.

$3rr$=1267500)7375000=Dividend the 2d.

6337500=$3rre$
48750=$3ree$
125=eee

6386375=Subducend.

$3rr$=128707500)988625000=Dividend 3d.

900952500=$3rre$
962850=$3ree$
343=eee

901915693 Subducend.

Remainder 86709307

This is carried to three Places of Decimals, and the Work is all the ſame as in whole Numbers, only making the Comma, to diſtinguiſh the Decimals, when you bring down the firſt three Cyphers to the Remainder.

If you have the Cube Root of any Fraction to find, reduce it into Decimals of 3, 6, 9, or 12 Places, as you deſire your Root to be leſs or more exact.

What is the Cube Root of $\frac{1}{2}$, the Work to three Places of Decimals, will ſtand in the following Manner:

,500000000),793 Root.
343

$3rr=14700$)157000 Dividend.

$132300=3rre$
$17010=3ree$
$729=eee$

150039 Subducend.

$3rr=1872300$)6961000=Dividend.

$5616900=3rre$
$21330=3ree$
$27=eee$

5638257 Subducend.

Remainder 1322743

To prove if your Work be true, Cube the Root, and to that Cube add the Remainder, and that Sum will be the ſame as the Number given to be extracted.

Proof of the laſt Example.

```
    ,793
    ,793
   -----
    2379
   7137
  5551
  -------
  ,628849 Square.
```

```
     ,628849
        ,793
    --------
     1886547
    5659641
   4401943
  -----------
   498677257 Cube.
     1322743 Reſt.
  -----------
  ,500000000 Proof.
```

But if the mixt Number or Fraction be commenſurable to its Root, then extract the Cube Root of the Numerator for the Numerator of the Root, and the Cube Root of the Denominator for the Denominator of the Root; ſo the Cube Root of $\frac{8}{27}$ will be $\frac{2}{3}$, for the Cube Root of 8 is 2, and of 27 is 3; which is $\frac{2}{3}$ and ſo of any other.

Queſtions to exerciſe the Cube Root.

IF a Bullet, whoſe Diameter is 4 Inches, weigh 9 Pound, what will a Bullet of the ſame Metal weigh, whoſe Diameter is 8 Inches?

RULE.

Like Solids are in triple Proportion to their homologous Sides, Diameters, Lines, &c.; it will be as the Cube of the Diameter is to its given Weight, ſo is the Cube of the other Diameter to the Weight ſought.

C.D. : ℔. :: C.D. : ℔.
If 64 : 9 :: 512 : 72. For 512×9=4608÷64=72 ℔.

If the Diameter of a Globe be 1 Inch, and the Solidity thereof be ,5238 of an Inch; what is the Solidity of another Globe, whoſe Diameter is 10 Inches?

C.D. : S. :: C.D. : S.
If 1 : ,5238 :: 1000 : 523,8 Inches. For ,5238×1000=523,8000÷1=523,8 the Solidity.

If a Globe of Silver of 1 Inch Diameter, be worth 14,4 Shillings, what will another Globe of Silver be worth, whoſe Diameter is 4 Inches?

If 1 : 14,4 :: 64 : 921,6. For 14,4×64=921,6÷1=921,6 Shillings, that is 46l. 1s. 7d$\frac{1}{5}$.

If a Saker of 3,75 Inches Diameter at the Bore, require 4℔. of Powder for its Charge; what will a Demi-Canon of 6,5 Inches in the Bore, require for its Charge?

C. D. ℔. C. D.
If 52,734375 : 4 :: 274,625 : 20,83.
For 274,625 × 4 = 1098,500 ÷ 52,734375 = 20,83 ℔. the Anſwer.

If a Braſs Saker, whoſe Diameter is 11,5 Inches, do weigh 1000 ℔. what will another Piece of Ordinance (of ths ſame Metal and Shape) weigh, whoſe Diameter is 8,75 Inches?

C. D. : ℔. :: C. D. ℔.
If 1520,875 : 1000 :: 669,921875 : 440,484

If a Ship, whoſe Burthen is 300 Tun, be 75 Feet in the Keel, what ſhall be the Burthen of another Ship (of the the ſame Mold) whoſe Keel is 100 Feet?

C. K. : Tun :: C. K. Tun.
If 421875 : 300 :: 1000000 : 711,111.

If a Bullet, whoſe Diameter is 4 Inches, weigh 9 ℔. what is the Diameter of another Bullet (of the ſame Metal) whoſe Weight is 72 ℔.? Anſwer 8 Inches. For,

℔. : C.D. ℔. C.D.
If 9 : 64 :: 72 : 512; whoſe Cube Root is 8 Inches the Diameter.

If the Diameter of a Globe be 1 Inch, and the Solidity ,5238 of an Inch, what is the Diameter of another Globe, whoſe Solidity is 523,8 Inches? Anſwer, 10 Inches. For,

Solid C. D. : : Solid : C.D.
If ,5238 : 1 : : 523,8 : 1000; whoſe Cube Root is 10 Inches.

If a Globe of Silver of 1 Inch Diameter be worth 14,4 Shillings, what will be the Diameter of another Globe of Silver that is worth 921,6 Shillings. Anſwer, 4 Inches. For,

S. C.D. S. C.D.
If 14,4 : 1 : : 921,6 : 64; whoſe Cube Root is 4.

If you would have the Side of a Cube = in Solidity to any given Solid, as Globe, Cylinder, Priſm, Cone, &c the Cube Root of the Solid Content will be the Side. But if you would have the Side double, treble in Quantity, × the Solidity by 2, 3, &c. if leſs, divide by 2, 4, &c. and the Cube Root of the Product or Quotient is the Anſwer.

Between 2 Numbers given, to find 2 mean Proportionals. *Rule*, × the leſs Extream by the Cube Root of the Quotient of the greater ÷ by the leſs; the Product is the leſſer of the 2 mean Proportionals, which × by the ſaid Cube Root, gives the greater Mean ſought.

Find 2 mean Proportionals betwixt 8 and 512. Then 512÷8=64, whoſe Cube Root is 4. And 8×4=32 the leſs Mean. And 32×4=128 the greater Mean.

Proof 8 : 32 :: 128 : 512.

Many more Uſes might be named, which the Learner will find by the Examples given, if they be underſtood.

The End of the Firſt Part.

P A R T II.

THE

ALGEBRAIST'S COMPANION.

Explanation of the Signs made Uſe of in this Algebra.

+	Signifies Addition, and is termed Plus, or more.
—	Signifies Subtraction, and is called Minus, or leſs.
×	Signifies Multiplication.
÷	Signifies Diviſion, or $\frac{24}{4}$, or $\frac{xxy}{n}$
@	Signifies Involution.
vv	Signifies Evolution, or the Root to be extracted.
::	Signifies Proportion.
∺	Signifies continued Proportion.
=	Signifies Equality, or equal to
√	Signifies the Root to be extracted.
C ▭	Signifies the Square muſt be compleated.

AXIOMS.

AXIOMS.

Firſt, If to equal Things are added equal Things, their Sums are equal.

Second, If from equal Things, equal Things are taken away, their Remainders will be equal.

Third, If equal Things be multiplied by equal Things, their Products will be equal.

Fourth, If equal Things be divided by equal Things, their Quotients will be equal.

Fifth, Things equal to one and the ſame Thing, are equal to one another.

* * *

ALGEBRA

IS a ſpecious Arithmetic, or an Arithmetic in Letters; it conſiſts of Addition, Subtraction, Multiplication, Diviſion, Involution, and Evolution, &c. or it is the Art of abſtract Reaſoning upon Quantity, by general and indefinite Repreſentations, in order to reſolve Problems, invent Theorems, and to demonſtrate both.

ADDITION.

Rule 1ſt. If Quantities are alike, and have the ſame Sign before them, to be added together, put down the Sum of the Coefficients with the common Sign before them, and the Common Letter after them.

EXAMPLES.

EXAMPLES.

	Steps.				
	1	x	$-5a$	$7x$	$3x+2y$
	2	$2x$	$-4a$	$3x$	$2x+5y$
1+2	3	$3x$	$-9a$	$10x$	$5x+7y$

N. B. 1+2 in the Margin, oppoſite the third Step, ſhew you, that the firſt and ſecond Steps are added together, and the Sum is placed againſt the third Step.

Rule 2d. If the Quantities to be added have different Signs before them, then put down the Difference of the two Coefficients, with the common Letter after it, and the Sign of the greater Quantity before it.

EXAMPLES.

	1	$4x$	$3b$	$-12a$	$-12xy$	$-7bc$	$+16$
	2	$-3x$	$3b$	$7a$	$16xy$	$+7bc$	-30
1+2	3	x	*	$-5a$	$4xy$	*	-14

To explain why an Affirmative Quantity added to a Negative, is made ſo much leſs, as the Negative is in Value, let us imagine a Perſon to have 40*x* Pounds; this Quantity repreſents the Value of his ready Money, Lands, Plate, Debts, &c. in his Poſſeſſion, or owing to him; this 40*x* would be the real Sum the Perſon is worth, if there were no Debts due from him to others. But let us ſuppoſe he owes

17*x*, which, tho' it be ſomething real, is yet of a direct contrary Nature to what is due to him, and muſt be expreſſed with a direct contrary Sign, and in the Nature of Debts, muſt deſtroy ſo much of the Perſon's Eſtate, as the Debt is in Value. Hence, if the Debts equal the Eſtate, they deſtroy each other, and the Perſon is worth nothing. If the Debts exceed the Eſtate, the Perſon is then worſe than nothing; but if the Eſtate exceeds the Debts, it is an Affirmative, and the Perſon's real Worth is the Difference.

Rule 3d. If the Quantities to be added be of different Kinds, and ſuch as will not incorporate, put them down in any Order, one after another, with their proper Signs before them; and this is all the Addition they are capable of. The Addition of compound Algebraick Quantities, is performed by collecting the ſeveral Members of every particular Species into as many Sums as there are Species; and then putting down the Sums in any Order, with their proper Sign before them.

EXAMPLES.

$$\begin{array}{r|c|llll}
 & 1 & x & a & 7m+12x+4dc \\
 & 2 & y & -b & 4a-20 \\
\hline
1+2 & 3 & x+y & a-b & 7m+12x+4dc+4a-20
\end{array}$$

$$\begin{array}{r|c|l}
 & 1 & 3xx+4abc-bb+20 \\
 & 2 & 2bb-3xx-2abc-15 \\
 & 3 & 2dd-2bb+abc+7 \\
\hline
1+2+3 & 4 & 3abc-bb+12+2dd
\end{array}$$

Subtraction

Subtraction of Algebra

IS performed by changing the Sign of every particular Member of the Quantities to be ſubtracted, *i. e.* making every negative Member affirmative, and every affirmative Member negative, and then adding it to the other, (or ſuppoſing them changed) for ſince it has been already obſerved, the ſubtracting any one Quantity from another, is the ſame in Effect as adding its contrary; and ſince changing the Sign of every Member to be ſubtracted, renders that Quantity juſt the contrary to what it was before, it is evident, that after this Change it may be added, and that the Sum of this Addition will be the true Difference.

EXAMPLES.

$$\begin{array}{r|l|cccc}
 & 1 & 2x & -2x & 12xy & 5a+12b \\
 & 2 & x & -x & 3xy & 2a+8b \\
\cline{3-6}
1-2 & 4 & x & -x & 9xy & 3a+4b
\end{array}$$

$$\begin{array}{r|l|cccc}
 & 1 & -10a & +4x & +7m & -8n \\
 & 2 & +10a & +10x & +8m & -8n \\
\cline{3-6}
1-2 & 3 & -20a & -6x & -m & *
\end{array}$$

Multiplication of Algebra.

RULE.

IF the Multiplicator and Multiplicand have Signs both alike, that is both affirmative, or both negative, the Product will then be affirmative. But if one be affirmative and the other negative, then the Product will be negative. Multiplication of ſimple Algebraick Quantities, is performed, firſt, by multiplying all the numeral Coefficients together, and then putting down after the Product, the Letters in both Factors, the Sign of the Product being prefixed to it. The Multiplication of compound Algebraick Quantities, is performed by multiplying every particular Member of the Multiplicator into all thoſe of the Multiplicand, and then reducing the whole into the leaſt Compaſs poſſible.

EXAMPLES.

	1	x	$4x$	$+4nr$	$-7m$	$-12xr$
	2	y	$7ym$	$-7y$	$+3n$	$-3m$
1×2	3	xy	$28xym$	$-28nry$	$-21mn$	$+36xrm$

	1	$x+m$
	2	$x+m$
1×x	3	$xx+mx$
1×m	4	$+mx+mm$
3+4	5	$xx+2mx+mm$

1×

	1	$4x-5a-2b$
	2	$3x-2a+5b$
$1\times 3x$	3	$12xx-15ax-6xb$
$1\times 2a$	4	$-8ax+10aa+4ab$
$1\times 5b$	5	$20xb-25ab-10bb$
$3+4+5$	6	$12xx-23ax+10aa+14xb-21ab+10bb$

Diviſion of Algebra

IS only the Reverſe of Multiplication, and all the Operations in this Rule may eaſily be performed, by conſidering that the Quotient multiplied by the Diviſor gives the Dividend.

Rule 1ſt. If the Quantities have like Signs, but no Coefficients, expunge the Quantities in the Dividend that are in the Diviſor, and put the Sign + before the Quotient.

EXAMPLES.

	1	xy	$-xy$	$am+bm$	$-ax-bx$
	2	x	$-x$	m	x
$1\div 2$	3	y	$+y$	$a+b$	$-a-b$

Rule 2d. If Quantities have Coefficients, divide them as in common Arithmetick, and expunge the Quantities as before.

EXAMPLES.

	1	$18ax$	$49xy$	$12am-24xm$
	2	$6x$	$-7x$	$6m$
1 ÷ 2	3	$3a$	$-7y$	$2a-4x$

Rule 3d. If the Diviſor and Dividend be the ſame, the Quotient is one.

EXAMPLES.

	1	xy	$9\,mr$	$7\,an+4\,nm$
	2	xy	$-9\,mr$	$7\,an+4\,nm$
1 ÷ 2	3	1	-1	1

Rule 4th. If the Quantities in the Diviſor are not like them in the Dividend, put them down as you do a vulgar Fraction.

EXAMPLES.

Divide xmy by nr; place it thus $\frac{xmy}{nr}$

Divide $5x+4y$ by $5n+r$; it will ſtand thus $\frac{5x+4y}{5n+r}$

If the Learner accuſtom himſelf to aſk theſe three Queſtions, it will make Diviſion eaſy. Firſt, what Sign × into the Diviſor, will give the Sign prefixed to the Dividend ; ſecondly, what numeral Coefficient of the Quotient, × into the numeral Coefficient of the Diviſor will make that of the Dividend ; and thirdly, what Letters × into thoſe of the Diviſor, will make thoſe of the Dividend ? The Signs, Coefficients, and Letters, which ariſe upon aſking theſe Queſtions, will be the true Quotient ſought, as well in compound as ſimple Algebraick Quantities.

EXAMPLES.

$-3a)12ab(-4b$

$12ab$

*

$-6xy)-24yxxx(+4xx$

$-24yxxx$

*

$2y+3x)16yyyy-72xxyy+81xxxx(8yyy-12xyy-18xxy$ [$+27x^3$

$16yyyy+24xyyy$

$-24xyyy-72xxyy$

$-24xyyy-36xxyy$

* $-36xxyy+81xxxx$

$-36xxyy-54xxxy$

* $+54xxxy+81xxxx$

$54xxxy+81xxxx$

* *

INVOLUTION

IS the Multiplication of like Quantities, or the raiſing of any given Quantity to any deſired Power.

EXAMPLES.

	1	x	xx	xxx	$xxxx$	$xxxxx$
	2	x	x	x	x	x
1 ⊚ 2	3	xx	xxx	$xxxx$	$xxxxx$	$xxxxxx$
or thus		x^2	x^3	x^4	x^5	x^6

This $a\times a=aa$, or a^2, is a Square. And $aa\times a=aaa$, or a^3, is a Cube. $xxx\times x=xxxx$, or x^4, is Biquadrate of x; or the fourth Power of x, &c.

	1	$x+y$	This is called a Binomial Root.
	2	$x+y$	
1 × x	3	$xx+xy$	
1 × y	4	$xy+yy$	
1 ⊚ 2	5	$xx+2xy+yy$	The Square of $x+y$.
		$x+y$	
5 × 5	6	$xxx+2xxy+xyy$	
5 × y	7	$xxy+2xyy+yyy$	
1 ⊚ 3	8	$xxx+3xxy+3xyy+yyy$.	The Cube of $x+y$.

	1	$x-y$.	This is called a reſidual Root.
	2	$x-y$	
1 × x	3	$xx-xy$	
1 × y	4	$-xy+yy$	
1 ⊚ 2	5	$xx-2xy+yy$.	The Square of $x-y$.
		$x-y$	
5 × x	6	$xxx-2xxy+xyy$	
5 × y	7	$-xxy+2xyy-yyy$	
1 ⊚ 3	8	$xxx-3xxy+3xyy-yyy$.	The Cube of $x-y$.

One may involve in this Manner to any Power whatever, by the Multiplication of any Power by its Root.

Let $x+y$ be involved to the 7th Power. The Powers from a Binomil have + before every Term, as you ſee by the Involution of $x+y$.

	1	$x^7+x^6+x^5+x^4+x^3+x^2+x$
	2	$y+y^2+y^3+y^4+y^5+y^6+y^7$
1 + 2	3	$x^7+x^6y+x^5y^2+x^4y^3+x^3y^4+x^2y^5+xy^6+y^7$,

now theſe joined, the 7th Power of $x+y$ ſtands in this Manner, without the Unicæ.

N. B. The leading Quantity x, decreaſes in Arithmetical Proportion; the other Quantity y, increaſes in the ſame Proportion.

To find the Unicæ, Sir **Isaac Newton** has this Method.

$$1\times\frac{m-0}{1}\times\frac{m-1}{2}\times\frac{m-3}{3}\times\frac{m-3}{4}\times\frac{m-4}{5}\times\frac{m-5}{6}$$

$$\times\frac{m-6}{7},\ \&c.$$

To explain this Theorem, *m* is the exponent of the Power, that is *m* is 7 in the 7th Power, 6 in the 6th Power, &c. The Meaning is this, $1\times7-0=7\div1=7$, the firſt Unicæ. Then $7\times7-1=6=7\times6=42$, and $42\div2=21$ the ſecond Unicæ. Then $21\times7-2=105\div3=35$, the third Unica. Then $35\times7-3=140\div4=35$, the fourth Unicæ. Then $35\times7-4=105\div5=21$, the fifth Unicæ. Then $21\times7-5=42\div6=7$, the 6th Unicæ, then $7\times7-6=7\div7=1$. Now the Unicæ being united to the above Powers, will ſtand in the following Manner.

$$x^7+7x^6y+21x^5y^2+35x^4y^3+35x^3y^4+21x^2y^5+7xy^6$$
$$+y^7$$

EVOLUTION.

AS Involution is a Multiplication, ſo Evolution is a Species of Diviſion. If the Power given have no Coefficient prefixed to it, and its Index can be divided by the Index of the Root required, the Quotient will be the Index of the Root ſought.

EXAMPLE.

EXAMPLE.

	1	x^6	$x^6 y^6$	$x^6 y^6 r^6$
1 vu 2	2	x^3	$x^3 y^3$	$x^3 y^3 r^3$
1 vu 3	3	x^2	$x^2 y^2$	$x^2 y^2 r^2$
1 vu 6	4	x	$x y$	$x y r$

N. B. The Figures 1 vu 2, 1 vu 3, &c. denote the Index of the Root to be extracted.

If the given Power have Coefficients, then you muſt extract their Roots, as in Arithmetic.

EXAMPLE.

	1	$81x^4$	$1296x^8 y^8$
1 vu 2	2	$9x^2$	$36x^4 y^4$
1 vu 4	3	$3x$	$6x^2 y^2$

But if the Root required cannot be truly extracted out of both the Coefficients and Indices of the given Power, then it is a Surd, and muſt have the Sign of the Root required prefixed to it.

EXAMPLE.

	1	x^5
1 vu 2	2	$\sqrt{x^5}$

To diſcover readily the Roots of all compound Powers in general, mind that if either the Sum, or difference of ſeveral Quantities be involved to any Power, there will ariſe ſo many ſingle Powers as there are different Quantities.

EXAMPLE.

$x+y+n$ ſquared, it will be as follows:

$$xx+2xy+2xn+yy+2ny+nn.$$

Here is the xx, yy, nn; ſo that one may conclude the Root is $x+y+n$, as the Signs are affirmative. When you have extracted the Root, involve it again to ſee whether it amounts to the ſame, if not, it is a Surd, or not right evolved.

Algebraick Fractions.

ALL Operations in Quantity are performed the ſame as in Numbers, ſo that an Inſtance or two of every Sort will be ſufficient.

REDUCTION.

TO change Fractions into one Denomination. Rule, Multiply the Denominators for a new Denominator, and every Numerator into all the Denominators but its own, for a new Numerator.

EXAMPLE.

$\frac{x}{y}+\frac{n}{r}=\frac{xr+ny}{yr}$ | $\frac{x}{y}\quad\frac{ny}{r}\quad\frac{2ae}{d}$ it is $\frac{xrd}{yrd}\quad\frac{nyyd}{yrd}\quad\frac{2aery}{yrd}$

$\frac{a}{x}\quad\frac{b}{3y}=\frac{3ay}{3xy}\quad\frac{bx}{3xy}$

To bring Integers into Fractions. Rule, Multiply Integers into the Denominator ſor a new Numerator, and under place the Denominator. Example, reduce $a+c$ to a Fraction, whoſe Denominator is d, it will be $\frac{da+de}{d}$.

To reduce any Quantity to the Form of an improper Fraction, as $x+y$, or x, draw a Line of Seperation, and under it write 1, then it will ſtand thus, $\frac{x+y}{1}$, or $\frac{x}{1}$.

To reduce Fractions to its, or their loweſt Terms. Rule, expunge the Quantities that are alike in both Numerator and Denominator, and put down the Remainder.

EXAMPLE.

$$\frac{12bcd}{4cd}=\frac{3bcd}{cd}=3b.$$

ADDITION.

$\frac{a}{b}+\frac{c}{d}$ the Sum is, $\frac{ad+cb}{bd}$. Add $\frac{p}{q}+\frac{x}{s}+\frac{t}{y}$ $+\frac{x}{u}$, the Sum is $\frac{psyu+xqyu+qstu+qsyx}{qsyu}$

SUBTRACTION.

From $\frac{p}{q}$ take $\frac{s}{r}$, it will be $\frac{pr-qs}{qr}$. $\frac{ab}{2x}-\frac{cd}{3f}$ $\frac{3abf-2cdx}{6xf}$.

MULTIPLICATION.

EXAMPLES.

$\frac{p}{q}\times\frac{r}{s}=\frac{pr}{qs}$ $x+\frac{y}{s}\times n+\frac{s}{r}=\frac{sx+y}{s}\times\frac{nr+s}{r}=$ $\frac{nrsx+nry+ssx+sy}{sr}$ Ex. $\frac{r}{s}\times\frac{s}{t}\times\frac{t}{u}\times\frac{u}{x}\times\frac{x}{y}=$ $\frac{rstux}{stuxy}=\frac{r}{y}$.

DIVISION.

DIVISION.

INvert your Diviſor, and then × as in Multiplication, and the Product will be the Quotient deſired.

EXAMPLE

Divide $\frac{r}{s}$ by $\frac{p}{q}$; place it thus: $\frac{q}{p}\Big)\frac{r}{s}\Big(\frac{qr}{ps}$

Divide $\frac{r}{s}+\frac{t}{u}$ by $\frac{a}{b}+\frac{c}{d}$. Reduce, and Invert, and it will ſtand thus:

$\frac{bd}{ad+bc}\Big)\frac{ru+st}{su}\Big(\frac{bdru+bdst}{adsu+bcsu}$ the true Quotient.

INVOLUTION.

INvolve the Numerator into itſelf for a new Numerator, and the Denominator into itſelf for a new Denominator.

EXAMPLE.

$\frac{a}{b}=\frac{aa}{bb}$, &c.

EVOLUTION.

Evolve the Numerators and Denominators.

EXAMPLE.

$$\frac{9xxyy}{4nn} = \frac{3xy}{2n}$$

N. B. If it has no Root prefix the Sign $\sqrt{}^{2}$ $\sqrt{}^{3}$ $\sqrt{}^{4}$, &c.

EXAMPLE

$$\frac{xxyy}{nnn} = \frac{xy}{\sqrt{nnn}}$$

Of SURDS.

THEY are ſuch Numbers as can't be exactly expreſſed in Figures, and are called irrational Numbers.

Their Reductions were difficult, but now are almoſt uſeleſs; and decimal Arithmetic has rendered them more capable of exact Expreſſion.

Addition and Subtraction.

IF they be Homogenal, add or ſubtract the rational Part, and to their Sum or Difference join the Surd.

ADDITION.

$$\begin{array}{r|c|lll} & 1 & 7\sqrt{xy} & 7x\sqrt{ac} & x\sqrt{aa+cc} \\ & 2 & 4\sqrt{xy} & 3x\sqrt{ac} & 4x\sqrt{aa+cc} \\ \hline 1+2 & 3 & 11\sqrt{xy} & 10x\sqrt{ac} & 5x\sqrt{aa+cc} \end{array}$$

SUBTRACTION.

$$\begin{array}{r|c|lll} & 1 & 14\sqrt{xy} & 12x\sqrt{cc} & 5y\sqrt{aa+cc} \\ & 2 & 7\sqrt{xy} & 7x\sqrt{cc} & 3y\sqrt{aa+cc} \\ \hline 1-2 & 3 & 7\sqrt{xy} & 5x\sqrt{cc} & 2y\sqrt{aa+cc} \end{array}$$

2d. If the Surds be Heterogeneal, add or ſubtract according to the Signs.

ADDITION.

$$\begin{array}{r|c|ll} & 1 & \sqrt{xy} & 4x\sqrt{aa} \\ & 2 & \sqrt{nx} & 3y\sqrt{zx} \\ \hline 1+2 & 3 & \sqrt{xy}+\sqrt{nx} & 4x\sqrt{aa}+3y\sqrt{xx} \end{array}$$

SUBTRACTION.

$$\begin{array}{r|c|l} & 1 & \sqrt{xy} \\ & 2 & \sqrt{nx} \\ \hline 1-2 & 3 & \sqrt{xy}-\sqrt{nx} \end{array}$$

Multiplication.

MULTIPLICATION.

IF the Surds be Homogenal, take away the Sign and it is done ; ſo the $\sqrt{xy}\times\sqrt{xy}=xy$ and $\sqrt{xx+aa}\times\sqrt{xx+aa}=xx+aa$.

	1	$\sqrt{x}$	$\sqrt{ba+da}$	$\sqrt{xx+yy}$
	2	$\sqrt{y}$	$\sqrt{ca}$	$\sqrt{xx-yy}$
1 × 2	3	$\sqrt{xy}$	$\sqrt{bcaa+cdaa}$	$\sqrt{xxxx-yyyy}$

If rational Quantities be joined to the Surds, then multiply the rational into the rational, and the Surd into the Surd, and join the Products together.

	1	$x\sqrt{mn}$	$6xy\sqrt{mn+y}$	$15\sqrt{xy}$
	2	$3y\sqrt{x}$	$3n\sqrt{r}$	$4\sqrt{n}$
1 × 2	3	$3xy\sqrt{mnx}$	$18xny\sqrt{mnr+ry}$	$60\sqrt{xny}$

Diviſion of Surd Quantities.

IF the Quantities are pure Surds of the ſame Kind, and can be divided off, without leaving a Remainder, divide them, and to their Quotient prefix their radical Sign.

EXAMPLES.

EXAMPLE.

$$\begin{array}{r|c|lll} & 1 & \sqrt{xy} & \sqrt{xynn+nybb} & \sqrt{xxxx-yyyy} \\ & 2 & \sqrt{x} & \sqrt{yn} & \sqrt{xx-yy} \\ \hline 1\div 2 & 3 & \sqrt{y} & \sqrt{xn+bb} & \sqrt{xx+yy} \end{array}$$

If Rational Quantities are joined to Surd Quantities of the ſame Kind, divide the Rational by the Rational, if it can be done, and to the Quotient join the Quotient of the Surd, divided by the Surd, with its firſt radical Sign.

EXAMPLES.

$$\begin{array}{r|c|ll} & 1 & 3dx\sqrt{xyn} & 20xyn\sqrt{ynxx+dcxx} \\ & 2 & 3d\sqrt{y} & 4y\sqrt{xx} \\ \hline 1\div 2 & 3 & x\sqrt{xn} & 5xn\sqrt{yn+dc} \end{array}$$

* * *

Involution of Surd Quantities.

WHEN the Surds are not joined to rational Quantities, they are involved to the ſame Height as their Index denotes, by taking away their radical Sign.

EXAMPLES.

$$\begin{array}{r|c|lll} & 1 & \sqrt{x} & \sqrt{xx+yy} & \sqrt{xx-yy} \\ 1 \circlearrowright 2 & 2 & x & xx+yy & xx-yy \end{array}$$

When Surds are joined to rational Quantities, involve the rational Quantities to the ſame Height as the Index of the Surd denotes; then multiply the involved Quantities into the Surd Quantities, after the radical Sign is taken away, as before.

EXAMPLES.

$$\begin{array}{r|c|ll} & 1 & x\sqrt{yy} & 5x\sqrt{xx+yy} \\ 1 \circledcirc 2 & 2 & xxyy & 25xxxx+25xxyy \end{array}$$

$$\begin{array}{r|c|l} & 1 & x : \sqrt[3]{xx+yy} \\ 1 \circledcirc 3 & 2 & xxxxx + xxxyy \end{array}$$

$$\begin{array}{r|c|l} & 1 & 3x : \sqrt[3]{xx+yy} \\ 1 \circledcirc 3 & 2 & 27xxxxx+27xxxyy \end{array}$$

EQUATIONS.

ALgebra ſolves Problems by Equations. In Equations we make uſe of five Proceſſes.

The firſt brings each Side of the Equation to an entire Fraction.

The ſecond, brings Equations into Integers, by multiplying alternately the Numerator of each Side into the Denominator of the other.

The third, brings by Tranſpoſition the ſeveral Members of the unknown Quantities to the ſame Side of the Equation, (*viz.*) to that Side, which after Tranſpoſition, will render them Affirmative.

The fourth, tranſpoſes all looſe Quantities from the Side of the unknown, to the other Side of the Equation.

The fifth, finds the unknown Quantity, by dividing the whole Equation by its Coefficient.

EXAMPLE.

	1	$\frac{2x}{3}+12=\frac{4x}{5}+6$
$\therefore$	2	$\frac{2x+36}{3}=\frac{4x+30}{5}$
2 × 3	3	$2x+36=\frac{12x+90}{5}$
3 × 5	4	$10x+180=12x+90$
4 — 10x	5	$180=2x+90$
5 — 90	6	$90=2x$
6 ÷ 2	7	$45=x$

	1	$\frac{3x}{4}+5=\frac{5x}{6}+2$
$\because$	2	$\frac{3x+20}{4}=\frac{5x+12}{6}$
2 × 4	3	$3x+20=\frac{20x+48}{6}$
3 × 6	4	$18x+120=20x+48$
4 — 18x	5	$120=2x+48$
5 —48	6	$72=2x$
6 ÷ 2	7	$36=x$

N. B. The three Dots in the Margin ſignify Conſequently.

EXAMPLE.

	1	$\frac{7x}{8} - 5 = \frac{9x}{10} - 8$
1×8	2	$7x - 40 = \frac{72x}{10} - 64$
2×10	3	$70x - 400 = 72x - 640$
$3 + 400$	4	$70x = 72x - 240$
$4 + 240$	5	$70x + 240 = 72x$
$5 - 70x$	6	$240 = 2x$
$6 \div 2$	7	$120 = x$

	1	$\frac{5x}{9} - 8 = 74 - \frac{7x}{12}$
$1 \quad \therefore$	2	$\frac{5x-72}{9} = \frac{888-7x}{12}$
2×9	3	$5x - 72 = \frac{7992-63x}{12}$
3×12	4	$60x - 864 = 7992 - 63x$
$4 + 63x$	5	$123x - 864 = 7992$
$5 + 864$	6	$123x = 8856$
$6 \div 123$	7	$x = 72$

	1	$\frac{x}{6} - 4 = 24 - \frac{x}{8}$
$\therefore$	2	$\frac{x-24}{6} = \frac{192-x}{8}$
1×6	3	$x - 24 = \frac{1152-6x}{8}$
3×8	4	$8x - 192 = 1152 - 6x$
$4 + 6x$	5	$14x - 192 = 1152$
$5 + 192$	6	$14x = 1344$
$6 \div 14$	7	$x = 96$

EXAMPLE.

EXAMPLES.

	1	$\frac{128}{3x-4} = \frac{216}{5x-6}$
$1 \times 3 - 4$	2	$128 = \frac{648x-864}{5x-6}$
$2 \times 5x - 6$	3	$640x - 768 = 648x - 864$
$3 + 864$	4	$640x + 96 = 648x$
$4 - 640x$	5	$96 = 8x$
$5 \div 8$	6	$12 = x$

	1	$\frac{xx-12}{3} = \frac{xx-4}{4}$
1×3	2	$xx - 12 = \frac{3xx-12}{4}$
2×4	3	$4xx - 48 = 3xx - 12$
$3 + 48$	4	$4xx = 3xx + 36$
$4 - 3xx$	5	$xx = 36$
5 vv 2	6	$x = \surd\ 36 = 6$

	1	$\frac{45}{2x+3} = \frac{57}{4x-5}$
$1 \times 2x + 3$	2	$45 = \frac{114x+171}{4x-5}$
$2 \times 4x - 5$	3	$180x - 225 = 114x + 171$
$3 + 225$	4	$180x = 114x + 396$
$4 - 114x$	5	$66x = 396$
$5 \div 66$	6	$x = 6$

EXAMPLE.

EXAMPLE.

	1	$\frac{42x}{x-2} = \frac{35x}{x-3}$
$1 \times x-2$	2	$42x = \frac{35xx-70x}{x-3}$
$2 \times x-3$	3	$42xx - 126x = 35xx - 70x$
$3 \div x$	4	$42x - 126 = 35x - 70$
$4 + 126$	5	$42x = 35x + 56$
$5 - 35x$	6	$7x = 56$
$6 \div 7$	7	$x = 8$

	1	$\frac{5xx}{16} - 8 = 12$
1×16	2	$5xx - 128 = 192$
$2 + 128$	3	$5xx = 320$
$3 \div 5$	4	$xx = 64$
4 ѵѵ 2	5	$x = \sqrt{64} = 8$

Of Quadratic Equations, and their Solutions.

THere are three Forms of Quadratic Equations, as follows:

1 $xx + 8x = 240$. This is the firſt Form.
2 $xx - 8x = 48$. The ſecond Form.

N. B. 8 is the Coefficient.

In either of theſe Forms, this is the practical Rule Take half the known Coefficient and ſquare it; and add that Square to both Sides of the Equation, and then the Forms will ſtand thus:

1 C□	3	$xx + 8x + 16 = 256$
1 C□	4	$xx - 8x + 16 = 64$

Now Evolve thoſe Steps, to find the Value of x.

3 ɯ 2	5	$x + 4 = \sqrt{256} = 16$
4 ɯ 2	6	$x - 4 = \sqrt{64} = 8$
5 − 4	7	$x = 12$
6 + 4	8	$x = 12$

And in this Manner you muſt proceed in all Queſtions belonging to either of thoſe Forms.

|1| $16x - xx = 48$. This is the third Form.

The Rule for this Form is this. Change all the Signs in the firſt Step on both Sides the Equation, (that is, what has the Sign + before it, muſt have the Sign —, and what — muſt have more) and then the Equation will ſtand thus, putting the Square of x to the left Hand.

1 chang'd	2	$xx - 16x = -48$
2 C□	3	$xx - 16x + 64 = 16$

N. B. 64 added to —48, gives the Sum 16.

3 ɯ 2	4	$x - 8 = \sqrt{16} = 4$.
4 + 8	5	$x = 12$

Algebraical Problems, and Queſtions.

No. 1.

WHAT Numbers are thoſe whoſe Difference is 9, and their Sum is 63?

	1	$x + 9 =$ the greater Number.
	2	$x =$ the leaſt.
$1 + 2$	3	$2x + 9 = 63$
$3 - 9$	4	$2x = 54$
$4 \div 2$	5	$x = 27$ the leaſt Number.
by the 1ſt.	6	$x + 9 = 36$ the greater Number.

Proof. $36 + 27 = 63$
$36 - 27 = 9$

No. 2.

What two Numbers are thoſe whoſe Difference is 8, and the Quotient of the greater divided by the leſs is 3?

	1	$x + 8 =$ the greater.
	2	$x =$ the leſs.
	3	$\frac{x + 8}{x} = 3$
$3 \times x$	4	$x + 8 = 3x$
$4 - x$	5	$2x = 8$
$5 \div 2$	6	$x = 4$ the leaſt Number.
by the 1ſt.	7	$x + 8 = 12$ the greater.

Proof. $12 - 4 = 8$
$12 \div 4 = 3$

No. 3.

No. 3.

What Number is that whoſe third Part exceeds its fourth Part by 9 ?

	1	x is the Number.
Then	2	$\frac{x}{3} = \frac{x}{4} + 9$
2×3	3	$x = \frac{3x}{4} + 27$
3×4	4	$4x = 3x + 108$
$4 - 3x$	5	$x = 108$ the Number.

Proof. $108 \div 3 = 36$
$108 \div 4 = 27$
And $36 - 27 = 9$

No. 4.

What Number is that whoſe third Part added to its fourth Part, the Number will be 35 ?

	1	x is the Number.
Then	2	$\frac{x}{3} + \frac{x}{4} = 35$
2×3	3	$x + \frac{3x}{4} = 105$
3×4	4	$4x + 3x = 420$
$\therefore$	5	$7x = 420$
$5 \div 7$	6	$x = 60$ The Number.

Proof. $60 \div 3 = 20$
$60 \div 4 = 15$
And $20 + 15 = 35$

No. 5.

What Number is that whoſe third Part — 4, is equal to its fourth Part + 3 ?

	1	x is the Number.
Then	2	$\frac{x}{3} - 4 = \frac{x}{4} + 3$
2×3	3	$x - 12 = \frac{3x}{4} + 9$
3×4	4	$4x - 48 = 3x + 36$
$4 + 48$	5	$4x = 3x + 84$
$5 - 3x$	6	$x = 84$

Proof. $84 \div 3 = 28$
$84 \div 4 = 21$
And $28 - 4 = 24$
And $21 + 3 = 24$

No. 6.

What two Numbers are thoſe whoſe Difference is 5, and the Difference of their Squares is 95 ?

	1	$x + 5$ is the greateſt Number.
	2	x is the leſs Number.
1 @ 2	3	$xx + 10x + 25$ the Square of the greater.
2 @ 2	4	xx the Square of the leſs.
$3 - 4$	5	$10x + 25 = 95$
$5 - 25$	6	$10x = 70$
$6 \div 10$	7	$x = 7$ the leſs
by the 1ſt.	8	$x + 5 = 12$ the greater.

Proof. $12 - 7 = 5$
And $12 \times 12 = 144$
And $7 \times 7 = 49$
Then $144 - 49 = 95$

No.

No. 7.

What two Numbers are thoſe whoſe Sum is 60, and the greater is to the leſs, as 3 to 2.

	1	$3x$ is the greater.
	2	$2x$ is the leſs.
$1+2$	3	$5x=60$
$3\div 8$	4	$x=12$
by the 1ſt.	5	$3x=36$ the greater Number.
by the 2d.	6	$2x=24$ the leaſt Number.

Proof. As $3:2::36:24$
For $2\times 36=72$
And $3\times 24=72$

No. 8.

What two Numbers are thoſe whoſe Sum is 56, and the greater is to the leſs, as 6 to 2 ?

	1	$6x=$ the greater.
	2	$2x=$ the leſs.
$1+2$	3	$8x=56$
$3\div 8$	4	$x=7$
by the 1ſt.	5	$6x=42$ the greater.
by the 2d.	6	$2x=14$ the leſs.

No. 9.

What two Numbers are thoſe whoſe Difference is 84, and the greater is to the leſs, as 4 to 1?

	1	$4x =$ the greater Number.
	2	$x =$ the leſs Number.
$1 - 2$	3	$3x = 84$
$3 \div 3$	4	$x = 28$ the leaſt.
by the 1ſt.	5	$4x = 112$ the greater.

Proof. As $4 : 1 :: 112 : 28$
For $1 \times 112 = 112$
And $4 \times 28 = 112$

No 10.

There is a Wall whoſe Length is double its Height, and its Height quintuple its Breadth, and it contains 1350 Cubic Feet, I demand the Length, Breadth, and Height of the Wall?

	1	$x =$ the Breadth.
	2	$5x =$ the Height.
	3	$10x =$ the Length.
$1 \times 2 \times 3$	4	$50xxx = 1350$
$4 \div 50$	5	$xxx = 27$
5 vv 3	6	$x = 3$ the Breadth.
by the 2d.	7	$5x = 15$ the Height.
by the 3d.	8	$10x = 30$ the Length.

Proof. $3 \times 15 = 45$
And $45 \times 30 = 1350$

No. 11.

We read, that formerly between the Mountain *Thornax*, and the Town *Halix*, there was a Trench dug, whoſe Length was thirty Times its Breadth, and the Breadth triple its Depth, and its whole Capacity is 138240 Cubick Feet; I demand its particular Dimenſions?

	1	$x =$ the Depth.
	2	$3x =$ the Breadth.
	3	$90x =$ the Length.
$1 \times 2 \times 3$	4	$270x^3 = 138240$
$4 \div 270$	5	$x^3 = 512$
4 vv 3	6	$x = 8$ Feet the Depth.
by the 2d.	7	$3x = 24$ Feet the Breadth.
by the 3d.	8	$90x = 720$ Feet the Length.

Proof. $8 \times 24 = 192$
And $192 \times 720 = 138240$

No. 12.

The Temple of *Æſculapius* was built in Form of a rectangled Triangle, one of whoſe Sides was 12 Paces, and the Sum of the other two was 36 Paces; I demand each Side ſeperately?

Let $s = 36$, and let $n = 12$

	1	$x =$ the Hypothenuſe of the Triangle.
	2	$s - x =$ the unknown Side.
	3	$n =$ the known Side.
1 ⊚ 2	4	xx
2 ⊚ 2	5	$ss - 2sx + xx$
3 ⊚ 2	6	nn
Then	7	$xx = ss - 2sx + xx + nn$ by the 47 of the firſt of Euclid.
$7 + 2sx$	8	$xx + 2sx = ss + xx + nn$
$8 - xx$	9	$2sx = ss + nn$
$9 \div 2s$	10	$x = \frac{ss+nn}{2s} = 20$ the Hypothenuſe.
by the 2d.	11	$s - x = 36 - 20 = 16$ the unknown Side.

Proof. $36 \times 36 = 1296 = ss$, and $12 \times 12 = 144 = nn$.

And $1296 + 144 = 1440 = ss + nn$; this divided by $72 = 2s$, gives 20 in the Quotient the Hypothenuſe.

And $20 \times 20 = 400$, the Square of the Hypothenuſe, $12 \times 12 = 144$, and $16 \times 16 = 256 + 144 = 400$

No. 13.

I am a brazen Lyon, my two Eyes, my Mouth, and the Sole of my right Foot, are ſo many ſeveral Pipes, they fill a Ciſtern, the right Eye fills it in two Days, the left in three, and the Sole of my Foot in 4, but my Mouth can fill it in 6 Hours; tell me in what Time they can fill the Ciſtern when they all run together?

1 $x =$ the required Time.

2 $\frac{x}{6}$ the Quantity carried into the Ciſtern by the Mouth.

3 $\frac{x}{48}$ the Quantity carried by the right Eye.

4 $\frac{x}{72}$ the Quantity carried by the left Eye.

5 $\frac{x}{96}$ the Quantity carried by the Sole of the right Foot.

Then by Inſpection we get the following Equation.

	6	$\frac{19x}{96} + \frac{x}{72} = 1$
6×96	7	$19x + \frac{96x}{72} = 96$
7×72	8	$1368x + 96x = 6912$
$\therefore$	9	$1464x = 6912$
$9 \div 1464$	10	$x = 4, \frac{44}{61} =$ the Time required.

No. 14.

What Number is that whoſe $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{6}$ exceeds itſelf by 1 ?

	1	$x =$ the Number. Then we have this
	2	Equation.
		$\frac{15x}{12} = x + 1$
2×12	3	$15x = 12x + 12$
$3 - 12x$	4	$3x = 12$
$4 \div 3$	5	$x = 4$. The Number ſought.

No. 15.

Three Perſons *A*, *B*, *C*, make a Contribution, which in the whole amounts to 76 Pounds, of which *A* contributes a certain Sum, *B* as much as *A*, and ten Pounds over, and *C* as much as *A* and *B* together; I demand their ſeveral Contributions?

	1	$x =$ *A*'s Contribution.
	2	$x + 10 =$ *B*'s Contribution.
$1 + 2$	3	$2x + 10 =$ *C*'s Contribution.
$1 + 2 + 3$	4	$4x + 20 = 76$
$4 - 20$	5	$4x = 56$
$5 \div 4$	6	$x = 14 =$ *A*'s Contribution.

Proof.	*A*'s Contribution	14
	B's $= 14 + 10 =$	24
	C's Contribution	38
		76

No. 16.

One hath ſix Sons, and every one is 4 Years older than his next younger Brother, and the oldeſt is three times as old as the youngeſt. I demand their ſeveral Ages?

	1	$x =$ the Age of the youngeſt.
	2	$x + 4$ the Age of the next.
	3	$x + 8$ the Age of the next.
	4	$x + 12 =$ the Age of the next.
	5	$x + 16 =$ the Age of the next.
	6	$x + 20 =$ the Age of the oldeſt.
Then	7	$3x = x + 20$
$7 - x$	8	$2x = 20$
$8 \div 2$	9	$x = 10$

No. 17.

It is required to divide the Number 84 into two Parts, that three times one Part, may be equal to 4 times the other?

	1	$4x =$ the greater.
	2	$3x =$ the leſs.
$1 + 2$	3	$7x = 84$
$3 \div 7$	4	$x = 12$
by the 1ſt.	5	$4x = 48$ the greater.
by the 2d.	6	$3x = 36$ the leſs.

No. 18.

It is required to divide the Number 60 into two Parts, that $\frac{1}{7}$ of the one may equal $\frac{1}{8}$ of the other?

	1	$8x =$ the greater Number.
	2	$7x =$ the leſs.
$1 + 2$	3	$15x = 60$
$3 \div 15$	4	$x = 4$
by the 1ſt.	5	$8x = 32$ the greater.
by the 2d.	9	$7x = 28$ the leſs.

No. 19.

One has a Leaſe for 33 Years, and being aſked how much of the Time was expired, anſwered, that $\frac{2}{3}$ of the Time paſt, was equal to $\frac{4}{5}$ of the Time to come. I demand the Time paſt and to come?

	1	$6x =$ the Time paſt.
	2	$5x =$ the Time to come.
$1 + 2$	2	$11x = 33$
$3 \div 11$	4	$x = 3$
by the 1ſt.	5	$6x = 18$ Time paſt.
by the 2d.	9	$5x = 15$ Years to come.

No. 20.

It is required to divide the Number 48 into two Parts, that one Part may be three times as much above 20, as the other wants of 20?

	1	$20+3x=$ the greater Part.
	2	$20-x=$ the leſs Part.
$1+2$	3	$40+2x=48$
$3-40$	4	$2x=8$
$4\div2$	5	$x=4$
by the 1ſt.	6	$20+3x=32$ the greater Part.
by the 2d.	7	$20-x=16$ the leſs Part.

No. 21

Two Perſons, *A* and *B* engage at Play; *A* has 72 Guineas, and *B* 52 before they begin, and after a certain Number of Guineas won and loſt between them, *A* goes away with three times as many as *B*. I demand the Number won and loſt?

	1	$x=$ the Guineas won and loſt.
	2	$52-x=$ *B*'s laſt Stock.
	3	$72+x=$ *A*'s laſt Stock.
Then	4	$72+x=156-3x$ this is *B*'s laſt Stock × by 3
$4+3x$	5	$72+4x=156$
$5-72$	6	$4x \quad 84$
$6\div4$	7	$x=21$ the Number of Guineas won and loſt.

No. 22.

Two Perſons, *A* and *B*, ſpeaking of their Crowns, ſays *A* to *B*, give me five of your Crowns, and I ſhall then have as much as you. Says *B* to *A*, give me five of your Crowns, and then I ſhall have three times as many as you. How many had each?

	1	$x =$ *A*'s Crowns.
	2	$x + 5 =$ *A*'s when he had received five of *B*.
	3	$x + 5 =$ *B*'s when he had given *A* 5.
	4	$x + 10 =$ *B*'s Crowns at firſt.
	5	$x - 5 =$ *A*'s Crowns when he had given *B* 5.
	6	$x + 15 =$ *B*'s Crowns when he had received 5 of *A*.
Then	7	$x + 15 = 3x - 15$
$7 + 15$	8	$x + 30 = 3x$
$8 - x$	9	$2x = 30$
$9 \div 2$	10	$x = 15$ the Crowns *A* had.
by the 4th	11	$x + 10 = 25$ the Crowns *B* had.

No. 23.

One meeting a Company of Beggars, gives each four Pence, and had ſixteen Pence left; but if he had given to each ſix Pence, he would have wanted twelve Pence. I demand the Number of Beggars?

	1	$x =$ the Number of Beggars.
Then	2	$4x + 16 = 6x - 12$
$2 + 12$	3	$4x + 28 = 6x$
$3 - 4x$	4	$2x = 28$
$4 \div 2$	5	$x = 14$ the Number of Beggars.

No.

No. 24.

One had three Debtors whoſe particular Sums he had forgot, but he finds from his Accompts, that *A*'s and *B*'s together, amounted to 38 Pounds, *B*'s and *C*'s to 52 Pounds, and *C*'s and *A*'s to 42 Pounds. I demand the Particulars ?

1 $x = A$'s.
Then 2 $38 - x = B$'s.
And 3 $14 + x = C$'s.
$1 + 3$ 4 $2x + 14 = 42 = A$' and C's.
$4 - 4$ 5 $2x = 28$
$5 \div 2$ 6 $x = 14 = A$'s Debt.
by the 2d. 7 $38 - x = 24 = B$'s Debt.
by the 3d. 8 $14 + x = 28 = C$'s Debt.

No. 25.

Find a Number which being multiplied by 12 and 48 added to the Product, as much may be produced as if the ſame Number was multiplied by 18 ?

1 $x =$ the Number ſought.
Then 2 $12x + 48 = 18x$
$2 - 12x$ 3 $6x = 48$
$3 \div 6$ 4 $x = 8$ the Number ſought.

No. 26.

A Man goes with a certain Sum of Money to a Tavern, where he borrows as much as he had then about him, and out of the whole ſpends eight Shillings; and thus he goes on to four Taverns ſucceſſively, borrowing as much Money as he brought thither, and out of the whole ſpending eight Shillings, till he has nothing left. What Money had he at firſt?

1 | $x =$ his Money at firſt.
2 | $2x =$ Money at firſt Tavern.
3 | $2x - 8 =$ his Money left.
4 | $4x - 16 =$ Money at ſecond Tavern.
5 | $4x - 24 =$ his Money left.
6 | $8x - 48 =$ Money at third Tavern.
7 | $8x - 56 =$ his Money left.
8 | $16x - 112 =$ Money at fourth Tavern.
Then | 9 | $16x - 120 = 0$
$9 + 120$ | 10 | $16x = 120$
$10 \div 16$ | 11 | $x = \frac{120}{16}$ 7*s.* 6*d.* his firſt Sum.

No. 27.

A Pack of Cards being laid into any Number of Heaps, ſo that the Spots on the under Card of each, added to the Number of Cards laid thereon, may make 12, by giving the Number of Heaps, and the Number of Cards left, to find all the Spots on the bottom Cards?

1 $6 = n$ the Number of Heaps.
2 $19 = r$ the Number of Cards leſt.
3 $52 = m$ the whole Pack.
4 $x =$ the Number of Spots required.

It is evident, that the Number of Spots on the bottom Card in each Heap, being added to the Number of Cards will make thirteen; therefore the Number of Spots and Cards together, in all the Heaps will be $13n$, from whence ſubtracting the Number of Spots, there will remain the Number of Cards only. So there ariſes this Equation.

5 $13n - x = m - r$.
6 $x = 13n - m + r = 45$ the No. of Spots.

No. 28.

A General diſpoſes his Men in a ſquare Form, and found he had 300 Men over; but when he thought of enlarging the Side of his Square by one Man, he found he ſhould want 349 Men for that Purpoſe. I demand the Number of Men in the Army?

	1	$x =$ the Side of the Square.
	2	$xx + 300 =$ the Army.
	3	$x + 1$ the Number of Men deſigned for the Side of the Square.
3 ⊛ 2	4	$xx + 2x + 1$
Then	5	$xx + 2x + 1 - 349 = xx + 300$
$5 + 349$	6	$xx + 2x + 1 = 649 + xx$
$6 - xx$	7	$2x + 1 = 649$
$7 - 1$	8	$2x = 648$
$8 \div 2$	9	$x = 324$ Men in the Side of the Square.

No.

No. 29.

It is required to divide 20 Shillings into 20 Pieces, conſiſting only of Sixpences and half Crowns?

	1	$x =$ the half Crowns.
Then	2	$20 - x =$ the Sixpences.
	3	$5x =$ the Number of Sixpences, into which the half Crowns may be reduced.
$2 + 3$	4	$4x + 20 = 40$ the Number of Sixpences in the whole.
$4 - 20$	5	$4x = 20$
$5 \div 4$	6	$x = 5$ the Half Crowns.
by the 2d.	7	$20 - x = 15$ the Sixpences.

No. 30.

Divide 24 Shillings into 24 Pieces, conſiſting only of Nine-pences and Thirteen-pence Halfpenny's?

	1	$x =$ the Ninepences.
	2	$24 - x =$ the Thirteen-pence Halfpenny's.
Then	3	$18x =$ the Halfpence the Nine-pences may be reduced to.
	4	$648 - 27x$ the Halfpence the Thirteen-pence Halfpenny's may be reduced to.
$3 + 4$	5	$648 - 9x = 576$ the Halfpence in 24 Shillings.
$5 + 9x$	6	$648 = 576 + 9x$
$6 - 576$	7	$72 = 9x$
$7 \div 9$	8	$x = 8$ the Nine-pences.
by the 2d.	9	$24 - x = 16$ the Thirteen-pence Halfpenny's.

No. 31.

What two Numbers are thoſe, that the greater is three times the leſs, and the Sum of their Squares is five times the Sum of their Numbers ?

	1	$3x =$ the greater.
	2	$x =$ the leſs.
$1 + 2$	3	$4x =$ their Sum.
1 ⊚ 2	4	$9xx$
2 ⊚ 2	5	xx
$4 + 5$	6	$10xx =$ the Sum of their Squares.
Then	7	$10xx = 20x$
$7 \div 10x$	8	$x = 2$ the leſs Number.
by the 1ſt.	9	$3x = 6$ the greater Number.

No. 32.

A Gentleman hires a Servant for a Year, or twelve Months, and gives him for his Wages ſix Pounds and a Livery Coat of a certain Value ; but at the End of ſeven Months the Servant obtains Leave to go away, and receives for his Wages fifty Shillings and the Livery Coat, which was his juſt Due. I demand the Value of the Coat ?

	1	$x =$ the Value of the Coat.
Then	2	$x + 120 =$ the Year's Wages.
And	3	$\frac{7x+840}{12} = x + 50.$
3×12	4	$7x + 840 = 12x + 600.$
$4 - 600$	5	$7x + 240 = 12x$
$5 - 7x$	6	$5x = 240$
$6 \div 5$	7	$x = 48$ Shillings, the Value of the Coat.

No. 33.

Two Perſons, *A* and *B* travel together, *A* with 100 l. *B* with 48 l. and they meet with a Company of Robbers, who take twice as much from *A* as from *B*, and lelft *A* three times as much as they left *B*. I demand what they took from each ?

	1	$x =$ the Money taken from *B*.
	2	$2x =$ the Money taken from *A*.
	3	$48 - x =$ the Money they left *B*.
	4	$100 - 2x =$ the Money they left *A*.
	5	$100 - 2x = 144 - 3x$
$5 + 3x$	6	$100 + x = 144$
$6 - 100$	7	$x = 44$ the Pounds taken from *B*.
bythe 2d.	8	$2x = 88$ the Pounds taken from *A*.

No. 34.

Divide the Number 90 into 4 ſuch Parts, that 2 being added to the firſt, ſubtracted from the ſecond, multiplied by the third, and dividing the fourth, may make them all equal.

	1	$x =$ the Number ſought.
	2	$x - 2 =$ the firſt.
	3	$x + 2 =$ the ſecond.
	4	$\frac{x}{2}$ the third.
	5	$2x =$ the fourth.
$2+3+4+5$	6	$4x + \frac{x}{2} = 90$
6×2	7	$8x + x = 180$
$7 \div 9$	8	$x = 20$ the Number ſought.

Proof.

Proof.

1 | $x-2=18$ which with two added $=20$
2 | $x+2=22$ when two is ſubtracted $=20$
3 | $\frac{x}{2}=10$ when multiplied by $2=20$
4 | $2x=40$ divided by $2=20$

$\overline{90}$

No. 35.

Suppoſe a Fiſh's Head was five Foot long, and his Tail was as long as his Head and half his Body, and his Body was as long as his Head and Tail. I demand the particular Dimenſions of it?

1 | $x=$ his Body.
2 | $\frac{x}{2}+5=$ his Tail.
3 | $5=$ his Head.
$2+3$ | 4 | $\frac{x}{2}+10=x$
4×2 | 5 | $x+20=2x$
$5-x$ | 6 | $20=x$ the Body.
bythe 2d. | 7 | $\frac{x}{2}+5=15$ the Tail.

No. 36.

The *Spartans* hired three famous Painters, *Arceſilaus*, *Euphranio*, and *Onaſia*, to adorn with Pictures the *Leſche Crotanorum*; they agreed to give *Arceſilaus* forty Shillings a Day, *Euphranio* was to have fifty, and *Onaſia* ſixty *per* Day, and the Work was finiſhed in 120 Days; and when they was paid for their Work, according to Agreement, they all three received equal Sums, I demand how many Days each Man worked according to this Proportion?

1 $x =$ *Onaſias*'s Time.
2 $\frac{6x}{5} =$ *Euphranio*'s Time.
3 $\frac{6x}{4} =$ *Arceſilaus*'s Time.
4 $\frac{74x}{20} = 120$
4 × 20 5 $74x = 2400$
5 ÷ 74 6 $x = 32\frac{16}{37}$ the Time *Onaſias* worked.
by the 2d. 7 $\frac{6x}{5} = 38\frac{34}{37}$ the Time *Euphranio* work'd
by the 3d. 8 $\frac{6x}{4} = 48\frac{24}{37}$ the Time *Arceſilaus* worked.

No. 37.

A Man and his Wife found by repeated Experience, that a Barrel of Beer which laſted them both twelve Days, would laſt him in her Abſence twenty Days. I demand how long it would laſt her in his Abſence?

1 $x =$ the Time it will laſt her.
2 20 Days the Time it will laſt him.
Then 3 $12 : 8 :: x : 20$
∴ 4 $8x = 240$
4 ÷ 8 5 $x = 30$ the Days it laſts her in his Abſence.

No.

No. 38.

A Vintner has two Sorts of Wines, which equally mixed, will be worth fifteen Pence a Quart, but if he mix them in the Proportion of 2 to 3, that is, after the Rate of two Quarts of his beſt to three of his worſt, the Mixture will then be worth but fourteen Pence the Quart. I demand the Price of each?

	1	$x=$ the Price of a Quart of the beſt.
	2	$30-x=$ the Price of a Quart of the worſt.
1×2	3	$2x=$ the Price of 2 Quarts of the beſt.
2×3	4	$90-3x=$ the Price of 3 Quarts of the worſt
$3+4$	5	$90-x=70$ Pence, the Price of 5 Quarts when mixed.
$5+x$	6	$90=70+x$
$6-70$	7	$x=20$ Pence, the Price of a Quart of the beſt.
by the 2d.	8	$30-x=10$ Pence, the Price of a Quart of the worſt.

No. 39.

There are two Places, *A* and *B*, 154 Miles diſtant from each other, from which two Perſons ſet out at the ſame Time, with a Deſign to meet one another, one travelling at the Rate of three Miles every two Hours, and the other at the Rate of five Miles every four Hours. I want to know how long and how far each travelled before they meet?

	1	$x =$ the No. of Hours each travelled.
	2	$\frac{3x}{2} =$ the Miles the firſt travelled.
	3	$\frac{5x}{4} =$ the Miles the ſecond travelled.
$2 + 3$	4	$\frac{11x}{4} = 154$
4×4	5	$11x = 616$
$5 \div 11$	6	$x = 56$ the Hours each travelled.

No. 40.

One ſets out from a certain Place, and travels 7 Miles in five Hours; and eight Hours after another ſets out from the ſame Place, and travels the ſame Road at the rate of five Miles in three Hours. I demand how long and how far the firſt travelled before the ſecond overtook him?

	1	$x =$ the Hours the firſt travelled.
	2	$x - 8 =$ the Hours the ſecond travelled.
	3	$\frac{7x}{5} =$ the Miles the firſt travelled.
	4	$\frac{5x-40}{3}$ the Miles the ſecond travelled.
Then	5	$\frac{5x-40}{3} = \frac{7x}{5}$
5×3	6	$5x - 40 = \frac{21x}{5}$
6×5	7	$25x - 200 = 21x$
$7 + 200$	8	$25x = 21x + 200$
$8 - 21x$	9	$4x = 200$
$9 \div 4$	10	$x = 50$ the Hours the firſt travelled.

by the 2d.	11	$x-8=42$ the Hours the ſecond travelled
by the 3d.	12	$\frac{7x}{5}=70$ Miles the firſt travelled.
by the 4th	13	$\frac{5x-40}{3}=70$ Miles the ſecond travelled.

No. 41.

Certain Noblemen made a Progreſs for their Pleaſure, every one of which carried the ſame Number of Pounds. The Number of Noblemen was equal to the Number of Servants each carried with him; the Number of Pounds each Nobleman carried was double the Number of Servants they all had amongſt them, and the Sum of all this Money was 3456 l. I demand the Number of Noblemen, Servants, and Pounds each carried?

	1	$x =$ the Number of Noblemen.
	2	$xx =$ the Number of Servants.
	3	$2xx =$ the No. of Pounds each carried.
$3 \times x$	4	$2xxx = 3456$
$4 \div 2$	5	$xxx = 1728$
5 vv 3	6	$x = 12$ the Number of Noblemen.
by the 2d.	7	$xx = 144$ the Number of Servants.
by the 3d.	8	$2xx = 288$ the Pounds each Nobleman carried.

No. 42.

There are two Numbers which are to each other as 3 to 2, and the Sum of their Cubes is 4375. *Quere*, the two Numbers?

	1	$3x$ = the greater Number.
	2	$2x$ = the leſs Number.
1 ⊚ 3	3	$27xxx$
2 ⊚ 3	4	$8xxx$
$3 + 4$	5	$35xxx = 4375$
$5 \div 35$	6	$xxx = 125$
6 ʋʋ 3	7	$x = 5$
by the 1ſt.	8	$3x = 15$ the greater Number.
by the 2d.	9	$2x = 10$ the leſs Number.

No. 43.

A Clock has two Hands turning upon the ſame Center, one turns round in 12 Hours, the other in 16 Hours. I demand the Synodical Period of theſe two Hands, that is, how many Revolutions each Hand makes before the ſwifter Hand retakes the ſlower?

N. B. After the two Hands ſet out from the ſame Place, the ſwifter Hand muſt make one Revolution more than the ſlower, before they can come together again.

	1	x = the No. of Hours in this Synodical Period.
	2	$\frac{x}{12}$ the Number of Revolutions the ſwifter Hand makes.
	3	$\frac{x}{16}$ the Number of Revolutions the ſlower Hand makes.
$2 - 3$	4	$\frac{x}{12} - \frac{x}{16} = 1$
4×12	5	$x - \frac{12x}{16} = 12$
5×16	6	$16x - 12x = 192$
∴	7	$4x = 192$

$7 \div 4$ | 8 | $x = 48$ the Hours in this Synodical Period.

by the 2d. | 9 | $\frac{x}{12} = 4$ the Number of Revolutions of the ſwifter Hand.

by the 3d. | 10 | $\frac{x}{16} = 3$ the Revolutions of the ſlower Hand.

No. 44.

A Perſon being aſked what o'Clock it was, anſwered, what was then paſſed from Noon, was equal to $\frac{3}{5}$ of what was remaining till Midnight. I demand the Hour?

| 1 | $x =$ the Time from Noon.

| 2 | $12 - x =$ the Time remaining to Midnight.

| 3 | $\frac{36 - 3x}{5} = \frac{3}{5}$ of the Time remaining.

Then | 4 | $x = \frac{36 - 3x}{5}$

4×5 | 5 | $5x = 36 - 3x$

$5 + 3x$ | 6 | $8x = 36$

$6 \div 8$ | 7 | $x = 4\frac{1}{2} =$ Hours from Noon.

by the 2d. | 8 | $12 - x = 7\frac{1}{2} =$ Hours to Midnight.

No. 45.

The Epicurean *Greeks* accounted their Thrium amongſt their moſt delicious Dainties. It was a Kind of Cake, or Wafer, of a determinate Weight, $\frac{1}{3}$ of it was of the fineſt Wheat Flower; $\frac{1}{6}$ of it was of Eggs, together with an Ounce and half of Lard, and the ſame of Honey; to theſe they added a Hemina of Milk, which contained 9 Ounces; theſe Things mixed in this Proportion, was baked upon a Fig-Leaf. I demand the Weight of the whole Cake, and the Weight of each Part?

	1	$x =$ the Weight of the whole Cake.
	2	$\frac{x}{3}$ the Weight of the Flower.
	3	$\frac{x}{6} =$ the Eggs.
	4	$3 =$ the Weight of Lard and Honey.
	5	$9 =$ the Weight of the Milk.
$2+3+4+5$	6	$\frac{6x}{12} + 12 = x$
6×12	7	$6x + 144 = 12x$
$7 - 6x$	8	$144 = 6x$
$8 \div 6$	9	$x = 24$ Ounces, the Weight of the whole Cake.

No 46.

A Captain being to maintain a Paſs with a certain Number of Men, of 416 Foot of Ground, placed his Men in a right Line, at equal Diſtances one from another, the Vacancies being ſufficient to receive no more than two Men a-piece; but finding by this Means he could defend but 250 Feet of Ground, he enlarged his Line, by opening his Vacancies twice as much as before, by which Means he found himſelf juſt able to make good the Paſs above-mention'd. I demand the Number of his Men?

	1	$x =$ the Number of Men.
	2	$x - 1 =$ the Number of Vacancies.
2×2	3	$2x - 2 =$ the Number of Men ſufficient to fill the Vacancies.

1 + 3	4	$3x-2=$ the Men ſufficient to fill the whole Line.
Then	5	$\frac{250}{3x-2}=$ the Quantity of Ground each Man ſtood upon.
3 × 2	6	$4x-4=$ the Men required to fill up the ſecond Vacancies.
1 + 6	7	$5x-4=$ the Men ſufficient to fill the whole Line.
Then	8	$\frac{416}{5x-4}$ the Quantity of Ground each Man ſtood on.
5 and 8	9	$\frac{416}{5x-4}=\frac{250}{3x-2}$
$9\times5x-4$	10	$416=\frac{1250x-1000}{3x-2}$
$10\times3x-2$	11	$1248x-832=1250x-1000$
$11+1000$	12	$1248x+168=1250x$
$12-1248x$	13	$2x=168$
$13\div2$	14	$x=84$ the Number of Men required.

No. 47.

One buys a certain Number of Eggs, $\frac{1}{2}$ of which he bought at two a Penny, and the other $\frac{1}{2}$ at three a Penny; all which he afterwards ſold at 5 for twopence, and contrary to his Expectation, loſt twelve Pence. I demand the Number he bought;

1	$x=$ the Number of Eggs bought.
2	$\frac{x}{2}=$ the Number at two a Penny.
3	$\frac{x}{2}=$ the Number at three a Penny.
4	$\frac{x}{4}=$ the Price of the former.

	5	$\frac{x}{6} =$ the Price of the latter.
	6	$\frac{2x}{5} =$ what he ſold them for.
$3 + 4$	7	$\frac{5x}{12} =$ what he gives for the whole.
Then	8	$\frac{2x}{5} + 12 = \frac{5x}{12}$
8×5	9	$2x + 60 = \frac{25x}{12}$
9×12	10	$24x + 720 = 25x$
$10 - 24x$	11	$x = 720$ the Number of Eggs bought.

No. 48.

One begun the World with a certain Sum of Money, which he improved ſo well by Trade, that at the End of the firſt Year he had doubled his firſt Stock, excepting 100 Pounds expended for the Uſe of his Family; the ſame he continues every Year, doubling his laſt Year's Stock, excepting 100 Pounds *per Ann.* as before, and at the End of 3 Years he found himſelf three times as rich as before he begun Trade. I demand his firſt Stock?

	1	$x =$ his firſt Stock.
	2	$2x - 100 =$ his Stock at the End of the firſt Year.
	3	$4x - 300 =$ his Stock at the End of the ſecond Year.
	4	$8x - 700 =$ his Stock at the End of the third Year.
Then	5	$8x - 700 = 3x$
$5 + 700$	6	$8x = 3x + 700$
$6 - 3x$	7	$5x = 700$
$7 \div 5$	8	$x = 140$ Pounds his firſt Stock-

No. 49.

One draws a certain Quantity of Wine out of a full Veſſel, which held 81 Gallons, and then filling it up with Water, takes a ſecond Draught of as much Wine and Water together, as before of Wine; and ſo he goes on taking an equal Quantity at every Draught, and ſtill recruiting the Veſſel with Water, whereby it happened that after the fourth Draught thus made, there remained but 16 Gallons of Wine in the Veſſel, all the reſt being Water. I demand how much he took out at a Draught?

N. B. As the whole Quantity of mixed Liquor before Drawing, is to the pure Wine before Drawing, ſo is the Quantity of mixed Liquor after Drawing to the Quantity of pure Wine after Drawing.

1 | $x =$ the Quantity of Wine in the Caſk after firſt Draught.
2 | $\frac{xx}{81}$ Wine after ſecond Draught.
3 | $\frac{xxx}{6561} =$ Wine after third Draught.
4 | $\frac{xxxx}{531441} =$ Wine after fourth Draught.
Then 5 | $\frac{xxxx}{531441} = 16$
5×531441 6 | $xxxx = 8503056$
6 vv 2 7 | $xx = 2916$
7 vv 2 8 | $x = 54 =$ the Wine in Caſk after firſt Draught.

No. 50.

Cupid complained that the Muſes had taken away his Arrows, *Clio* ſaith he hath taken $\frac{1}{5}$, *Euterpe* $\frac{1}{12}$, *Thalia* $\frac{1}{8}$, *Erato* $\frac{1}{7}$, *Melpomene* $\frac{1}{20}$, *Terpſichore* $\frac{1}{4}$, *Polyhymnia* 30, *Urania* 105, and *Calliope*, the moſt ſpightful of them all, would have 360; ſo that now I have but 5 left. How many had he at firſt?

	1	$60x =$ the Number of his Arrows.
Then	2	$\frac{60x}{7} + \frac{60x}{8} + 500 = 25x$
2×7	3	$60x + \frac{420x}{8} + 3500 = 175x$
3×8	4	$480x + 420x + 28000 = 1400x$
$\therefore$	5	$1400x = 900x + 28000$
$5 - 900x$	6	$500x = 28000$
$6 \div 500$	7	$x = 56$
by the 1ſt.	8	$60x = 3360$ the No. of his Arrows.

No. 52.

Two Men ſpeaking of their Pounds, ſays *A* to *B*, my Pounds are in Proportion to yours as 3 to 2: Yes, ſays *B*, and the Sum of our Pounds, is in Proportion to the Sum of the Squares of our Pounds as 1 to 13. How many Pounds had each?

	1	$3x =$ *A*'s Pounds.
	2	$2x =$ *B*'s Pounds.
1 ⊚ 2	3	$9xx =$ the Square of *A*'s.
2 ⊚ 2	4	$4xx =$ the Square of *B*'s.
$3 + 4$	5	$13xx =$ the Sum of their Squares.
$1 + 2$	6	$5x =$ the Sum of their Pounds.

Then	7	$5x : 13xx :: 1 : 13$
$\therefore$	8	$13xx = 65x$
$8 \div 13x$	9	$x = 5$
by the 1ſt.	10	$3x = 15 =$ *A's* Pounds.
by the 2d.	11	$2x = 10 =$ *B's* Pounds.

No. 52.

A General who had fought a Battle, upon reviewing his Army, whoſe Foot was thrice the Number of his Horſe, finds that before the Battle $\frac{1}{12} - 120$ of his Foot had deſerted, and of his Horſe $\frac{1}{20} + 120$, beſides $\frac{1}{4}$ of his whole Army was ſent into Gariſons, (reckoning the Sick and Wounded) and $\frac{3}{8}$ of his Army remained, the reſt being either ſlain or taken Priſoners; now if you add 3000 to the Slain, the Sum will be equal to half the Foot he had at the Beginning. What Number of Men were in the Army?

	1	$60x =$ Foot.
	2	$20x =$ Horſe.
$1 + 2$	3	$80x =$ the whole Army.
	4	$5x - 120$ the Number of Foot that deſerted.
	5	$x + 120$ the Number of Horſe that deſerted.
	6	$20x =$ the Men in Gariſon.
	7	$30x$ the Number that remained.
$4+5+6+7$	8	$56x$
$3 - 8$	9	$24x + 3000 = 30x$

9—

$9 - 24x$	10	$6x = 3000$
$10 \div 6$	11	$x = 500$
by the 1ſt.	12	$60x = 30000 =$ the No. of his Foot.
by the 2d.	13	$20x = 10000 =$ the No. of his Horſe.
$12 + 13$	14	$80x = 40000 =$ the whole Army.

No. 53.

One bought three Books, whoſe Prices were in Proportion as 12, 5, and 1; now if the Price of the firſt be doubled, of the ſecond trebled, and of the third quadrupled, the Sum of theſe Products will as much exceed fifty Shillings, as the Sum of the Prices of the greateſt and middle is below twenty-five Shillings. *Quere*, what did each coſt?

	1	$12x =$ the Price of the greateſt.
	2	$5x =$ the Price of the Middle.
	3	$x =$ the Price of the leaſt.
1×2	4	$24x$
2×3	5	$15x$
3×4	6	$4x$
$4+5+6$	7	$43x$
$1 + 2$	8	$17x$
Then	9	$43x - 50 = 25 - 17x$
$9 + 50$	10	$43x = 75 - 17x$
$10 + 17x$	11	$60x = 75$
$11 \div 60$	12	$x = 1\frac{1}{4}$ Shillings $=$ 1*s.* 3*d.* the Price of the leaſt.
by the 2d.	13	$5x = 6\frac{1}{4}$ Shillings $=$ 6*s.* 3*d.* the Price of the ſecond.
by the 1ſt.	14	$12x = 15$ Shillings the Price of the greateſt.

No.

No. 54.

Find a Number, which if added to itſelf, and the Sum multiplied by the ſame, and the ſame Number ſtill ſubtracted from the Product; and laſtly, the Remainder divided by the ſame, that it may produce 13?

	1	$x =$ the Number ſought.
$1 + x$	2	$2x$
$2 \times x$	3	$2xx$
$3 - x$	4	$2xx - x$
$4 \div x$	5	$\frac{2xx - x}{x} = 13$
$5 \times x$	6	$2xx - x = 13x$
$6 + x$	7	$2xx = 14x$
$7 \div 2x$	8	$x = 7$ the Number ſought.

No. 55.

Divide 36 into two ſuch Parts that if 12 be added to the firſt, and 6 to the ſecond, the former may be double the Sum of the latter?

	1	$x =$ the greater.
Then	2	$36 - x =$ the leſs.
$1 + 12$	3	$x + 12$
$2 + 6$	4	$42 - x$
Then	5	$\frac{x + 12}{2} = 42 - x$
5×2	6	$x + 12 = 84 - 2x$
$6 + 2x$	7	$3x + 12 = 84$
$7 - 12$	8	$3x = 72$
$8 \div 3$	9	$x = 24$ the greater.
by the 2d.	10	$36 - x = 12$ the leſs.

No. 56.

Divide 36 into two ſuch Parts, that if the leſs be multiplied by 12, and the greater made more by 6, the Multiplication will be twice the Addition?

	1	$x =$ the leaſt Part.
Then	2	$36 - x =$ the greater Part.
1×12	3	$12x$
$2 + 6$	4	$42 - x$
$3 \div 2$	5	$6x = 42 - x$
$5 + x$	6	$7x = 42$
$6 \div 7$	7	$x = 6$ the leaſt Part.
by the 2d.	8	$36 - x = 30$ the greater Part.

No. 57

Find the Side of a Cube whoſe Superfices is to the Solidity, as 6 to 11?

	1	$x =$ the Side of the Cube.
	2	$6xx =$ the Superfices.
	3	$xxx =$ the Solidity.
	4	$6 : 11 :: 6xx : xxx$
$\therefore$	5	$6xxx = 66xx$
$5 \div xx$	6	$6x = 66$
$6 \div 6$	7	$x = 11$ the Side of the Cube.

No. 58.

There is an Army to which if you add $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ of itſelf, and take away 5000, the Men remaining will be 100000. What is the Number of the Army?

	1	$12x =$ the Army.
	2	$6x = \frac{1}{2}$ the Army.
	3	$8x = \frac{2}{3}$ of the Army.
	4	$9x = \frac{3}{4}$ of the Army.
$1+2+3+4$	5	$35x$
Then	6	$35x = 105000$
$6 \div 35$	7	$x = 3000$
by the 1ſt.	8	$12x = 36000$ the No. of the Army.

No. 59.

Find the Side of a Square whoſe Area is to the Sum of the Sides, as 45 to 12 ?

	1	$x =$ the Side of the Square.
Then	2	$4x =$ the Sum of the Sides.
And	3	$xx =$ the Area.
Then	4	$45 : 12 :: xx : 4x$
$\therefore$	5	$12xx = 180x$
$5 \div 12x$	6	$x = 15$ the Side of the Square.

No. 60.

A Perſon being aſked how old he was, anſwered, if I quadruple $\frac{2}{3}$ of my Years, and add $\frac{1}{2}$ of them $+ 50$ to the Product, the Sum will be ſo much above 100, as the Number of my Years is now below 100. What was his Age ?

	1	$6x =$ his Age.
Then	2	$16x = \frac{2}{3}$ of his Age quadrupled.
And	3	$3x + 50 = \frac{1}{2}$ his Age and 50
$2 + 3$	4	$19x + 50$
$4 - 100$	5	$19x - 50 = 100 - 6x$

$5 + 6x$	6	$25x - 50 = 100$
$6 + 50$	7	$25x = 150$
$7 \div 25$	8	$x = 6$
by the 1ſt.	9	$6x = 36$ his Age.

No. 61.

One being aſked what Hour of the Day it was, anſwered, the Day at this Time is 16 Hours long, now if $\frac{1}{2}$ the Hours paſt, be added to $\frac{2}{3}$ of the Remainder, you will have the Hour deſired, reckoning from Sun-riſing. What was the Hour?

	1	$x =$ the Hours paſt.
Then	2	$16 - x =$ the Hours remaining.
And	3	$\frac{x}{2} + \frac{32 - 2x}{3} = x$
3×2	4	$x + \frac{64 - 4x}{3} = 2x$
4×3	5	$3x + 64 - 4x = 6x$
$5 + 4x$	6	$10x = 3x + 64$
$6 - 3x$	7	$7x = 64$
$7 \div 7$	8	$x = 9\frac{1}{7}$ the Time paſt.
by the 2d.	9	$16 - x = 6\frac{6}{7}$ the Time remaining.

No. 62.

Divide the Number 50 into two Parts, ſo that the greater Part being divided by 7, and the leſs multiplied by 3, the Sum of this Product, and the former Quotient may make the ſame Number propoſed, which was 50. *Quere* the Numbers?

1	$7x =$ the greater Part.
2	$50 - 7x =$ the leſs Part.

Then

Then	3	$x + 150 - 21x = 50$
$3 + 21x$	4	$21x + 50 = x + 150$
$4 - 50$	5	$21x = x + 100$
$5 - x$	6	$20x = 100$
$6 \div 20$	7	$x = 5$
by the 1ſt.	8	$7x = 35$ the greater Part.
by the 2d.	9	$50 - 7x = 15$ the leſs Part.

No. 63.

If a Man gains 30 Crowns a Week, how much muſt he ſpend a Week to have 500 Crowns together, with the Expence of four Weeks remaining at the Year's End?

	1	$x =$ his weekly Expence.
	2	$52x =$ his yearly Expence.
Then	3	$56x + 500 = 1560$
$3 - 500$	4	$56x = 1060$
$4 \div 56$	5	$x = 18\frac{13}{14}$.

No. 64.

A Labourer, after 40 Weeks in which he had been at Work, lays up 28 Crowns, leſs the Pay of three Weeks, and finds he had expended 36 Crowns more the Pay of 11 Weeks. What Pay did he receive per Week?

	1	$x =$ his weekly Pay.
	2	$40x =$ his whole Wages.
Then	3	$28 - 3x + 36 + 11x = 40x$
$3 + 3x$	4	$28 + 36 + 11x = 43x$
$4 - 11x$	5	$64 = 32x$
$5 \div 32$	6	$x = 2$ Crowns his weekly Wages.

No. 65.

A Son aſked his Father how old he was, his Father anſwered, if you take 5 from my Years, and divide the Remainder by 8, the Quotient will be $\frac{1}{3}$ of your Age; but if you add 2 to your Age, and multiply the whole by 3, and then ſubtract 7 from the Product, you will have the Number of the Years of my Age. *Quere*, the Age of each?

	1	$8x+5=$the Father's Age.
	2	$x=\frac{1}{3}$ of the Son's Age.
	3	$3x=$the Son's Age.
Then	4	$9x-1=8x+5$
$4+1$	5	$9x=8x+6$
$5-8x$	6	$x=6$
by the 1ſt.	7	$8x+5=53$ Years the Age of the Father.
by the 3d.	8	$3x=18$ the Son's Age.

No. 66.

Find two Numbers, the Product whereof is 240, and triple of the greater divided by the leſs is 5?

	1	$5x=$the greater.
	2	$3x=$the leſs.
1×2	3	$15xx=240$
$3\div 15$	4	$xx=16$
4 υν 2	5	$x=4$
by the 1ſt.	6	$5x=20$ the greater Number.
by the 2d.	7	$3x=12$ the leſs Number.

No. 67.

Find two Numbers, to the Sum if you add 6 the whole ſhall be double the greater; and if you ſubtract 2 from their Difference, the Remainder will be half the leaſt?

	1	$2x=$ the Sum
Then	2	$x+3=$ the greater Number.
1 — 2	3	$x-3=$ the leaſt.
Then	4	$4=$ the Difference leſs $2=$ half the leaſt.
And	5	$8=$ the leaſt Number.
5 and 4	6	$14=$ the greater Number.

No. 68.

Two Men have a Mind to purchaſe a Houſe valued at 1200 l. ſays *A* to *B*, if you give me $\frac{2}{3}$ of your Pounds I can pay for the Houſe alone; but, ſays *B* to *A*, if you give me $\frac{3}{4}$ of your Pounds, I can pay for it. How many Pounds had each?

	1	$4x=$ *A*'s Pounds.
	2	$3x=$ *B*'s Pounds.
Then	3	$6x=1200$
$3 \div 6$	4	$x=200$
by the 1ſt.	5	$4x=800$ Pounds $=$ *A*'s Money.
by the 2d.	6	$3x=600$ Pounds $=$ *B*'s Money.

No. 69.

Divide 100 twice into two Parts, ſo that the major Part of the firſt Diviſion may be three Times the minor Part of the ſecond Diviſion; and the major Part of the ſecond, may be double the minor Part of the firſt?

	1	$3x=$ major Part of the firſt Diviſion.
	2	$100-3x=$ minor Part of the firſt.
	3	$100-x=$ major Part of the ſecond Diviſion.
	4	$x=$ minor Part of the ſecond Diviſion.
Then	5	$100-x=200-6x$
$5+6x$	6	$100+5x=200$
$6-100$	7	$5^{x}=100$
$7 \div 5$	8	$x=20$ the minor Part of the ſecond Diviſion.
by the 3d.	9	$10-x=80$ the major Part of the ſecond Diviſion.
by the 2d.	10	$100-3x=40$ minor Part of the firſt Diviſion.
by the 1ſt.	11	$3x=60$ major Part of the firſt Diviſion.

No. 70.

Find three Numbers, ſo that the firſt and $\frac{1}{2}$ the Remainder, the ſecond and $\frac{1}{3}$ the Remainder, and the third and $\frac{1}{4}$ the Remainder, may always make 34? Let x, y, and z, be the Numbers.

	1	$x+\frac{y+z}{2}=34$
	2	$y+\frac{x+z}{3}=34$
	3	$z+\frac{x+y}{4}=34$
1×2	4	$2x+y+z=68$
2×3	5	$3y+x+z=102$
3×4	6	$4z+x+y=136$
$5-4$	7	$2y-x=34$
5×4	8	$4z+4x+12y=408$
$8-6$	9	$3x+11y=272$

$7+x$	10	$x+34=2y$
10×3	11	$3x+102=6y$
$9-11$	12	$11y-102=272-6y$
$12+6y$ &c.	13	$17y=374$
$13\div 17$	14	$y=22$ the ſecond Number.
by the 10th.	15	$x=10$ the firſt Number.
by the 4th.	16	$z=26$ the third Number.

No. 71.

Three Merchants, from three ſeveral Fairs, meet together at an Inn, where they reckon up their Gains, and find them amount to 780 l. moreover, if you add the Gain of the firſt and ſecond, and ſubtract the Gain of the third from the Sum, you will have the Gain of the firſt + 82 l.; but if you add the Gain of the ſecond and third, and from the Sum ſubtract the Gain of the firſt, there remains the Gain of the third, —43 l. What was the Gain of each?

	1	$x=$ third Man's Gain.
Then	2	$698-2x=$ the firſt Man's Gain.
	3	$780=$ their whole Gain.
$1+2$	4	$698-x=$ the Sum of the firſt and third Man's Gain.
$3-4$	5	$82+x-$ the ſecond Man's Gain.
$1+5$	6	$2x+82=$ the Gain of ſecond and third.
$6-2$	7	$4x-616=x-43$
$\therefore$	8	$3x=573$
$8\div 3$	9	$x=191$ Pounds the third Man's Gain.
by the 2d.	10	$698-2x=316$ Pounds the Gain of the firſt Man.
by the 5th.	11	$82+x=273$ Pounds the ſecond Man's Gain.

No. 72.

A Hare being 50 of her own Leaps before a Dog, makes 4 Leaps to the Dogs three; but two Leaps of the Dogs are as much as three Leaps of the Hare's. How many Leaps muſt the Dog take to catch the Hare?

	1	$4x+50=$ all the Hare leaps.
	2	$3x=$ all the Dogs leaps.
		Dogs : Hares : : Dogs.
Then	3	$2 : 3 :: 3x : \frac{9x}{2}$
$\therefore$	4	$\frac{9x}{2} = 4x + 50$
4×2	5	$9x=8x+100$
$5 - 8x$	6	$x=100$
by the 2d.	7	$3x=300=$ all the Dogs leaps.
by the 1ſt.	8	$4x+50=450=$ all the Hare leaps.

No. 73.

In three Bags is a certain Quantity of Pounds Sterling. The Sum in the firſt and ſecond Bag is twenty Pounds; the Sum of the Pounds in the ſecond and third Bag is forty-eight Pounds; and the Sum of the Pounds in the firſt and third Bag is 44 Pounds; what Number of Pounds is in each Bag? Let x, y, and z be the Bags.

	1	$x+y=20$
	2	$y+z=48$
	3	$x+z=44$
$2 - 3$	4	$y-x=4$
$1 - 4$	5	$2x=16$

5 ÷ 2

$5 \div 2$	6	$x=8$
$1 - 6$	7	$y=12$
$2 - 7$	8	$z=36$

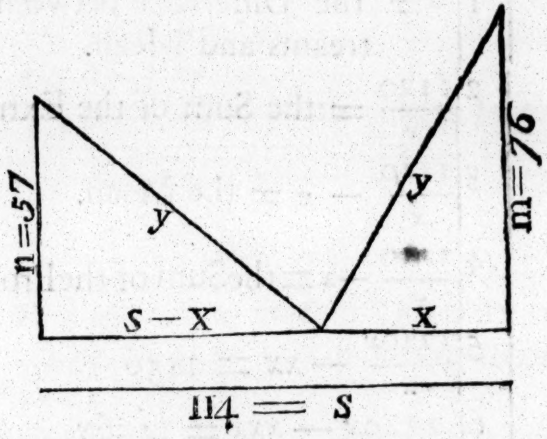
No. 74.

There are three Numbers in Geometrical Proportion, and ſuch, if the Difference between the Sum of the Extreams and the Mean be multiplied by the Sum of the Extreams, the Product will be 1120; but if the ſaid Differences be multiplied by the Sum of all three Proportionals, the Product will be 1456. What are the Numbers?

	1	$x=$ the Difference between the Extreams and Mean.
Then	2	$\frac{1120}{x}$ = the Sum of the Extreams.
And	3	$\frac{1120}{x} - x =$ the Mean.
$2 + 3$	4	$\frac{2240}{x} - x =$ theSum of theProportionals
$4 \times$ 1ſt,	5	$\frac{2240x}{x} - xx = 1456$
$5 \times x$	6	$2240x - xxx = 1456x$
$6 + xxx$	7	$2240x = 1456x + xxx$
$7 \div x$	8	$xx + 1456 = 2240$
$8 - 1456$	9	$xx = 784$
9 vv 2	10	$x = 28$ the Difference betwixt the Extreams and Mean.
by the 2d.	11	$\frac{1120}{x} = 40$ the Sum of Extreams.
by the 3d.	12	$\frac{1120}{x} - x = 12$ the Mean.

No. 75.

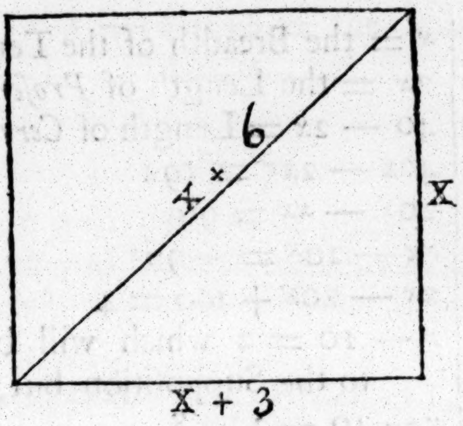
At *Thebes*, in the Street that was called *Prætides*, ſtood the Temple of *Euclia*, 76 Foot high, oppoſite to which ſtood the Temple of *Boedromius-Apollo*, which was 57 Foot high, and the Diſtance betwixt them was 114 Foot. Between theſe two Temples was placed the Statue of a Lyon cut in Marble, equi-diſtant from the Top of each Temple, which was ſaid to be dedicated to *Hercules*, when he had defeated *Erchimus*, King of the *Orcaomenians*, I want the Diſtance the Lyon ſtood from the Baſe of each Temple?



	1	$x =$ the leaſt Diſtance.
	2	$mm + xx = yy$ per Figure.
	3	$nn + ss - 2sx + xx = yy$ per Figure.
2 and 3	4	$mm = nn + ss - 2sx$
$4 + 2sx$	5	$mm + 2sx = nn + ss$
$5 - mm$	6	$2sx = nn + ss - mm$
$6 \div 2s$	7	$x = \frac{nn + ss - mm}{2s} = 45\frac{11}{12}$ the leaſt Diſtance.
And	8	$144 - x = 68\frac{1}{12}$ the greateſt.

No. 76.

The Temple of the three Graces at *Athens* ſtood upon a rectangled *Area*, whoſe Length exceeded its Breadth by three Paces, and the Diſtance from one Angle to that which was diametrically oppoſite to it, exceeded the Length alſo by three Paces. I demand its Length, Breadth, and diametrical Diſtance?



	1	$2xx + 6x + 9 = xx + 12x + 36$ per Figure.
1 ±	2	$xx - 6x = 27$
2 C ☐	3	$xx - 6x + 9 = 36$
3 vv 2	4	$x - 3 = 6$
4 + 3	5	$x = 9 =$ Breadth.
And	6	$x + 3 = 12 =$ Length.
And	7	$x + 6 = 15$ diametrical Diſtance.

No. 77.

Buporthmos was a Promontory of *Peloponneſus*, running into the Sea, not far from the Iſland *Fricana*, oppoſite to the Gulph of *Scylla*; on this Promontory ſtood a Temple forty Paces long, divided into two unequal Areas; the greater was dedicated to *Ceres*, the leſs to her Daughter *Proſerpine*; the Capacity of *Ceres*'s was 192 ſquare Paces, *Proſerpine*'s was twice as long as it was broad. What was the Breadth of this Temple?

	1	$x =$ the Breadth of the Temple.
	2	$2x =$ the Length of *Proſerpine*'s.
Then	3	$40 - 2x =$ Length of *Ceres*'s.
3 × x	4	$40x - 2xx = 192$
4 ÷ 2	5	$20x - xx = 96$
Then	6	$xx - 20x = -96$
6 C ▭	7	$xx - 20x + 100 = 4$
7 vu 2	8	$x - 10 = 2$ which will be contrary to the Suppoſition but, it will be
	9	$x = 10 - 2 = 8$

No. 78.

In the City of *Megara*, in the Way through *Jupiter*'s Wood to the Caſtle *Caria*, were to be ſeen two little ſquare Temples, dedicated to *Nectelius-Bacchus* and *Apoſtrophia-Venus*; their Pavements were laid with Stone a Foot ſquare, but the Side of *Venus*'s Temple exceeded that of *Bacchus*'s by 12 Foot, and both the Pavements taken together, contained 2120 Stones. I demand the Length of each ſeperately?

	1	$x =$ the Side of *Bacchus*'s Temple.
	2	$x+12=$the Side of *Venus*'s Temple.
1 ⊛ 2	3	$xx=$the Area of *Bacchus*'s Temple.
2 ⊛ 2	4	$xx+24x+144=$ the Area of *Venus*'s Temple.
3 + 4	5	$2xx+24x+144=2120$
5 ÷ 2	6	$xx+12x+72=1060$
6 — 72	7	$xx+12x=988$
7 *C* □	8	$xx+12x+36=1024$
8 *vu* 2	9	$x+6=32$
9 — 6	10	$x=26=$ the Side of *Bacchus*'s Temple.
by the 2d.	11	$x+12=38=$ the Side of *Venus*'s Temple.

No. 79.

There are 480 Men to be placed in an Oblong, whoſe Length and Breadth together make 52. How many is each?

	1	$x=$the No. of Men in Length.
	2	$y=$the No. of Men in Breadth.
Then	3	$x+y=52$
And	4	$xy=480$
3 ⊛ 2	5	$xx+2xy+yy=2704$
4 × 4	6	$4xy=1920$
5 — 6	7	$xx-2xy+yy=784$
7 *vu* 2	8	$x-y=28$
3 + 8	9	$2x=80$
9 ÷ 2	10	$x=40$
3 — 10	11	$y=12$

No. 80.

TwoCaptains diſtribute each of them 1200Crowns among a certain Number of Soldiers; one has 40 Soldiers more than the other, and its found that thoſe of the leſſer Number received five Crowns a Man more than thoſe of the greater Number. How many Soldiers had each Captain?

	1	x=the leſſer Number.
	2	$x+40$=the greater Number.
Then	3	$\frac{1200}{x}$ = Each Man's Share in the leſs Number.
And	4	$\frac{1200}{x+40}$ = Each Man's Share in the greater Number.
$\therefore$	5	$\frac{1200}{x+40}+5=\frac{1200}{x}$
$5 \times x+40$	6	$1200+5x+200=\frac{1200+48000}{x}$
$6 \times x$	7	$1200x+5xx+200x=1200x+48000$
$7-1200x$	8	$5xx+200x=48000$
$8 \div 5$	9	$xx+40x=9600$
9 C □	10	$xx+40x+400=10000$
10 vv 2	11	$x+20=100$
$11-20$	12	$x=80$=the leſſer Number of Men.
by the 2d.	13	$x+40=120$ the greater Number of Men.

No. 81.

Two Parties of Soldiers have each of them an equal Number of Crowns to be diſtributed amongſt them; in one Party there were four Men more than in the other, the Money being divided, each Man in the leſs Party had eight Crowns more than thoſe of the greater, and the Number of Crowns to be diſtributed contained 172 more than the Number of Men in both Parties together. How many Men are in each Party, and what was the Number of Crowns?

	1	$x =$ the Number of Men in the leſs Party.
	2	$x + 4 =$ the Number in the greater Party.
$1 + 2$	3	$2x + 4 =$ the Number in both Parties.
$3 + 172$	4	$176 + 2x =$ the Number of Crowns.
Then	5	$\frac{176+2x}{x+4} =$ the Share of one Man in the greater Party.
And	6	$\frac{176+2x}{x} =$ the Share of one Man in the leſs Party.
∴	7	$\frac{176+2x}{x+4} + 8 = \frac{176+2x}{x}$
$7 \times x + 4$	8	$\frac{176x+2xx+8x+704}{x} = 176+2x+8x+32$
$8 \times x$	9	$176x + 2xx + 8xx + 32x = 176x + 2xx+8x+704$
∴	10	$8xx + 24x = 704$
$10 \div 8$	11	$xx + 3x = 88$

11 C □	12	$xx + 3x + 2,25 = 90,25$
12 vv 2	13	$x + 1,5 = \sqrt{90,25} = 9,5$
13 — 1,5	14	$x = 8$ the Number of Men in the leſs Party.
by the 2d.	15	$x + 4 = 12$ the Number of Men in the greater Party.
by the 4th.	16	$176 + 2x = 192$ the Number of Crowns each Party had.

No. 82.

There is a certain Square whoſe Side is 110 Inches, now it is required to find the Dimenſions of a rectangled Parallelogram, whoſe Perimeter ſhall be greater by four Inches, and whoſe Area ſhall be leſs by four ſquare Inches, than that of the above-mentioned Square?

	1	$12100 =$ the Area of the Square.
	2	$12096 =$ the Area of the Parallelogram.
	3	$440 =$ the Perimiter of the Square.
	4	$444 =$ the Perimeter of the Parallelogram.
Then	5	$222 = \frac{1}{2}$ the Perimeter.
	6	$x =$ the Length of the Parallelogram.
	7	$222 - x =$ the Breadth.
7 × 6	8	$222x - xx = 12096$ the Area.
	9	$xx - 222x = -12096$
9 C □	10	$xx - 222x + 12321 = 225$
10 vv 2	11	$x - 111 = 15$
Then	12	$x = 126$ the Length of the Parallelogram.
by the 7th.	13	$222 - x =$ the Breadth.

No. 83.

Find a Number, to the Quadruple of which, if you add 91, the whole ſhall be to the Square of the Number ſought as 3 to 4?

	1	$x =$ the Number.
	2	$4x + 91$
Then	3	$4x + 91 : xx :: 3 : 4$
$\therefore$	4	$3xx = 16x + 364$
$4 - 16x$	5	$3xx - 16x = 364$
$5 \div 3$	6	$xx - \frac{16x}{3} = \frac{364}{3}$
$6\ C\ \Box$	7	$xx - \frac{16x}{3} + \frac{64}{9} = \frac{364}{3} + \frac{64}{9} = \frac{1156}{9}$
$7\ \omega\ 3$	8	$x - \frac{8}{3} = \sqrt{\frac{1156}{9}} = \frac{34}{3}$
$8 + \frac{8}{3}$	9	$x = \frac{42}{3} = 14$ the Number.

No. 84.

Two Merchants have a Parcel of Silk, the firſt 40 Ells, the ſecond 90; the firſt ſells for a Crown $\frac{1}{3}$ of an Ell more than the ſecond. When the Sale was over, they had taken between them 42 Crowns. How many Ells did each of them ſell for a Crown?

1 | $120 =$ the thirds of an Ell the firſt had.

2 | $270 =$ the thirds of an Ell the ſecond had.

3 | $x + 1 =$ the thirds of an Ell the firſt ſells for a Crown.

4 | $x =$ the thirds of an Ell the ſecond ſells for a Crown,

Then

Then	5	$42=\frac{120}{x+1}+\frac{270}{x}$ *per* Queſtion
$5\times x+1$	6	$42x+42=120+\frac{270x+270}{x}$
$6\times x$	7	$42xx+42x=120x+270x+270$
$\therefore$	8	$42xx+42x=390x+270$
$8-390x$	9	$42xx-348x=270$
$9\div 42$	10	$xx-\frac{348x}{42}=\frac{270}{42}$
10 C □	11	$xx-\frac{348x}{42}+\frac{30276}{1764}=\frac{270}{42}+\frac{30276}{1764}$ $=\frac{41616}{1764}$
$11\ \omega\ 2$	12	$x-\frac{174}{42}=\sqrt{\frac{41616}{1764}}=\frac{204}{42}$
$12+\frac{174}{42}$	13	$x=\frac{378}{42}=9$, the thirds of an Ell the ſecond ſells.
by the 3d.	14	$x+1=10$, the thirds of an Ell the firſt ſells.

No. 85.

A Man buys a Piece of Linen, and by ſelling it again, he gains 12 Shillings, $-\frac{1}{10}$ of what he bought it for; and finds by this Means that he had gained as much for 100 Shillings as the Linen coſt him. What Price was the Linen bought and ſold at?

	1	$10x=$ the Shillings it coſt.
Then	2	$10x:12-x::100:\frac{1200-100x}{10x}$ *per* Queſtion.
$\therefore$	3	$10x=\frac{1200-100x}{10x}$

$3 \times 10x$	4	$100xx = 1200 - 100x$
$4 + 100x$	5	$100xx + 100x = 1200$
$5 \div 100$	6	$xx + x = 12$
6 C □	7	$xx + x + ,25 = 12,25$
7 vv 2	8	$x + ,5 = \sqrt{12,25} = 3,5$
$8 - ,5$	9	$x = 3$
by the 1ſt.	10	$10x = 30$ Shillings it coſt.
∴	11	$10x + 9 = 39$ Shillings he ſold it for.

No. 86.

Two Country Women *A* and *B*, carry 100 Eggs to Market, and in the Sale of them, one took as much Money as the other; but *A*, who had the beſt Eggs, ſays to *B*, had I carried as many Eggs as you, I ſhould have had eighteen Pence for them; *B* replies, if I had brought as many Eggs as you, I ſhould have had but eight Pence for them. How many Eggs had each?

	1	$x =$ the Number *A* carried.
	2	$100 - x =$ the Number *B* carried.
Then	3	$x : 8 :: 100 - x : \frac{800-8x}{x}$
And	4	$100 - x : 18 :: x : \frac{18x}{100-x}$
∴	5	$\frac{18x}{100-x} = \frac{800-8x}{x}$
$5 \times x$	6	$\frac{18xx}{100-x} = 800 - 8x$
$6 \times 100 - x$	7	$18xx = 80000 - 1600x + 8xx$
$7 - 8xx$	8	$10xx = 80000 - 1600x$
$8 \div 10$	9	$xx = 8000 - 160x$
$9 + 160x$	10	$xx + 160x = 8000$
10 C □	11	$xx + 160x + 6400 = 14400$

11 vv

11 vv 2	12	$x + 80 = \sqrt{14400} = 120$
12 — 80	13	$x = 40$ = *A*'s Eggs.
by the 2d.	14	$100 - x = 60$ = *B*'s Eggs.

No. 87.

A General is to place 969 Men, ſo that there may be 40 more in Depth than in Front. How many muſt there be in each ?

	1	$x =$ the Men in Front.
	2	$x + 40 =$ the Men in Depth.
2 × 1ſt.	3	$xx + 40x = 969$
3 C □	4	$xx + 40x + 400 = 1369$
4 vv 2	5	$x + 20 = 37$
4 — 20	6	$x = 17$ the Men in Front,
by the 2d.	7	$x + 40 = 57 =$ the Men in Depth.

No. 88.

A Merchant bought a certain Number of Yards for 70 Shillings, and found that if there had been 4 Yards more, every Yard would have ſtood him in two Shillings leſs than it did. I demand the Number of Yards ?

1	$x =$ the Number of Yards he bought
2	$x + 4 =$ the Number had he had four Yards more.
3	$\frac{70}{x}$ = the Price of a Yard by the firſt Suppoſition.
4	$\frac{70}{x+4} =$ the Price of the ſecond.

Then

Then	5	$\frac{70}{x} - 2 = \frac{70}{x+4}$
$5 \times x$	6	$70 - 2x = \frac{70x}{x+4}$
$6 \times x + 4$	7	$70x + 280 - 2xx - 8x = 70x$
$7 - 280$	8	$70x - 2xx - 8x = 70x - 280$
$8 - 70x$	9	$-2xx - 8x = -280$
$9 \div -2$	10	$xx + 4x = 140$
10 C□	11	$xx + 4x + 4 = 144$
11 ω 2	12	$x + 2 = 12$
$12 - 2$	13	$x = 10$ = the Number of Yards he bought.

No. 89.

One bought a Horſe, and ſold him again for 56 Pounds, and gained as much per Cent. as the Horſe coſt. I demand the Price of the Horſe?

	1	x = the Price of the Horſe.
	2	56 = the Pounds he ſold for.
	3	$56 - x$ = the Gain.
Then	4	$x : 56 - x :: 100 : \frac{5600 - 100x}{x}$
And	5	$x = \frac{5600 - 100x}{x}$
$5 \times x$	6	$xx = 5600 - 100x$
$6 + 100x$	7	$xx + 100x = 5600$
7 C□	8	$xx + 100x + 2500 = 8100$
8 ω 2	9	$x + 50 = 90$
$9 - 50$	10	$x = 40$ = the Pounds the Horſe coſt.

No. 90.

There are two Numbers whoſe Difference is 15, and if the Product be divided by 2, the Quotient will give the Cube of the leſs Number. What are the Numbers ?

	1	$x =$ the leſs Number.
Then	2	$15 + x =$ the greater:
And	3	$xxx = \frac{xx+15x}{2}$
3×2	4	$2xxx = xx + 15x$
$4 \div x$	5	$2xx = x + 15$
$5 - x$	6	$2xx - x = 15$
$6 \div 2$	7	$xx - \frac{x}{2} = 7{,}5$
7 C □	8	$xx - \frac{x}{2} + {,}0625 = 7{,}5625$
8 vu 2	9	$x - {,}25 = 2{,}75$
$9 + {,}25$	10	$x = 3$
by the 2d.	11	$x + 15 = 18$

No. 91.

There are three Numbers in geometrical Proportion, the Difference of the Extreams is 16, and the Mean is 6. I demand the Extreams ?

	1	$x =$ one of the Extreams.
	2	$x + 16 =$ the other.
Then	3	$xx + 16x = 36$
3 C □	4	$xx + 16x + 64 = 100$
4 vu 2	5	$x + 8 = 10.$
$5 - 8$	6	$x = 2$
by the 2d.	7	$x + 16 = 18$

No. 92.

One buys 120 ℔. of Pepper, and as many of Ginger, and received for a Crown one Pound of Ginger more than of Pepper; ſo that the whole Price of the Pepper came to ſix Crowns more than the Price of the Ginger. How many Pounds of each did he buy for a Crown?

	1	$x =$ the Pounds of Pepper for a Crown.
	2	$x + 1 =$ the Pounds of Ginger for a Crown.
	3	$\frac{120}{x}$ the Price of the Pepper.
	4	$\frac{120}{x+1}$ the Price of the Ginger.
Then	5	$\frac{120}{x+1} + 6 = \frac{120}{x}$
$5 \times x+1$	6	$120 + 6x + 6 = \frac{120x+120}{x}$
$6 \times x$	7	$120x + 6xx + 6x = 120x + 120$
$7 - 120x$	8	$6xx + 6x = 120$
$8 \div 6$	9	$xx + x = 20$
9 C □	10	$xx + x + ,25 = 20,25$
10 υυ 2	11	$x + ,5 = 4,5$
$11 - ,5$	12	$x = 4$
by the 2d.	13	$x + 1 = 5$

No. 93.

A Man buys 80 ℔. of Pepper, and 36 ℔. of Saffron; ſo that for eight Crowns he had 14 Pounds of Pepper more than he had of Saffron for 26 Crowns, and what he laid out amounted to

188 Crowns. How many Pounds of Pepper had he for 8 Crowns, and how many of Saffron for 26?

	1	$x =$ Saffron.
	2	$x + 14 =$ Pepper.
Then	3	$x : 26 :: 36 : \frac{936}{x}$
And	4	$x + 14 : 8 :: 80 : \frac{640}{x+14}$
$\therefore$	5	$\frac{640}{x+14} + \frac{936}{x} = 188$
$5 \times x + 14$	6	$640 + \frac{936x + 13104}{x} = 188x + 2632$
$6 \times x$	7	$640x + 936x + 13104 = 188xx + 2632x$
$7 - 1576x$	8	$188xx + 1056x = 13104$
$8 \div 188$	9	$xx + 5{,}617x = 69{,}702$
$9\ C\ \square$	10	$xx + 5{,}617x + 7{,}884 = 77{,}586$
10 vv 2	11	$x + 2{,}808 = 8{,}808$
$11 - 2{,}808$	12	$x = 6 =$ Pounds of Saffron.
by the 2d.	13	$x + 14 = 20$ Pounds of Pepper.

No. 94.

Some Companions at an Inn ſpent 3 l. 15 s. before paying the Reckoning, two went away, by which each of the reſt was obliged to pay ten Shillings more than his equal Club. I demand the Number at firſt?

	1	$x =$ the Number at firſt.
Then	2	$\frac{75}{x} =$ every Man's true Club.
And	3	$x - 2 =$ the Number of Men when two were gone.

	4	$\frac{75}{x-2} =$ Each Man's Reckoning now.
∴	5	$\frac{75}{x-2} - 10 = \frac{75}{x}$
$5 \times x-2$	6	$75 - 10x + 20 = \frac{75x-150}{x}$
$6 \times x$	7	$75x - 10xx + 20x = 75x - 150$
$7 - 75x$	8	$-10xx + 20x = -150$
∴	9	$10xx - 20x = 150$
$9 \div 10$	10	$xx - 2x = 15$
10 *C* □	11	$xx - 2x + 1 = 16$
11 uv 2	12	$x - 1 = 4$
$12 + 1$	13	$x = 5$ the Number of Men at firſt.

No. 95.

Two Travellers ſet out at the ſame Time from two Cities, the one from *A*, and the other from *B*, which are 120 Miles diſtant from one another; the firſt goes five Miles a Day, and the other 3 Miles leſs than the Number of Days in which they meet. When will they meet?

	1	$x =$ the Time required.
	2	$5x =$ the Miles the firſt travelled.
	3	$x - 3 =$ the Second's daily Journey.
3×1	4	$xx - 3x =$ the Miles the ſecond travelled.
$2 + 4$	5	$xx + 2x = 120$
5 *C* □	6	$xx + 2x + 1 = 121$
6 uv 2	7	$x + 1 = 11$
$7 - 1$	8	$x = 10$ the Days they travel.

No. 96.

A Poſt ſets out from *A* towards *B*, who travels 8 Miles a Day; after he had got 27 Miles another ſets out from *B* to meet him, who goes every Day $\frac{1}{20}$ of the whole Diſtance of the Places *A* and *B*, and meets the firſt Poſt after ſo many Days as is $\frac{1}{20}$ of the ſaid Diſtance. I demand the Number of Miles between them?

	1	$x =$ the Diſtance required.
	2	$\frac{x}{20} =$ the Time the ſecond travelled.
Then	3	$1 : 8 :: \frac{x}{20} : \frac{8x}{20}$
And	4	$\frac{8x}{20} + 27$ the Miles the firſt travelled.
Then	5	$1 : \frac{x}{20} :: \frac{x}{20} : \frac{xx}{400}$
And	6	$\frac{xx}{400} =$ the Miless the ſecond travelled.
$4 + 6$	7	$\frac{xx}{400} + \frac{8x}{20} + 27 = x$
7×400	8	$xx + \frac{3200x}{20} + 10800 = 400x$
8×20	9	$20xx + 3200x + 216000 = 8000x$
$9 - 3200x$	10	$20xx + 216000 = 4800x$
$10 - 216000$	11	$20xx = 4800x - 216000$
$11 - 4800x$	12	$20xx - 4800x = -216000$
$12 \div 20$	13	$xx - 240x = -10800$
13 C □	14	$xx - 240x + 14400 = 3600$
14 ω 2	15	$x - 120 = 60$
$15 + 120$	16	$x = 180$ the Diſtance of the two Places.

No. 97.

Two Men depart at the ſame Time, one from *London* the other from *Lincoln*, keeping the ſame Road, and when they meet, ſays *A* to *B*, I find I have travelled 20 Miles more than you, and have gone as much in 6 $\frac{2}{3}$ Days as you have done hitherto; I know, ſays the other, you are the beſt Walker, but if I go on as I have done, I ſhall finiſh my Journey in 15 Days hence. What was the Diſtance of the Places, and how much did they travel per Day?

	1	$x + 20 =$ the Miles gone by *A*, when they meet.
	2	$x =$ what *B* goes in the ſame Time.
1 + 2	3	$2x + 20 =$ the whole Diſtance.
		Thirds
Then	4	$x : 20 : x+20 : \frac{20x+400}{x}$
		Thirds
And	5	$x+20 : 45 :: x : \frac{45x}{x+20}$
∴	6	$\frac{45x}{x+20} = \frac{20x+400}{x}$
6 × x	7	$\frac{45xx}{x+20} = 20x + 400$
7 × x + 20	8	$45xx = 20xx + 800x + 8000$
8 — 20xx	9	$25xx = 800x + 8000$
9 ÷ 25	10	$xx = 32x + 320$
10 — 32x	11	$xx - 32x = 320$
11 C □	12	$xx - 32x + \underline{256} = 576$
12 vu 2	13	$x - 16 = \sqrt{576} = 24$
13 + 16	14	$x = 40$
by the 1ſt.	15	$x + 20 = 60$

14

14 + 15	16	$2x + 20 = 100$ the whole Diſtance in Miles.
And	17	*A* goes 6 Miles per Day, and *B* 4.

No. 98.

A deſires *B*, to let him know how many Pounds he had in his Purſe; *B* anſwered if I double the Number, and add 20 more than the ſquare Root of the Sum doubled, to the doubled Sum, and laſtly add this Sum to its Square, the Number of my Pounds would then be 194040. Now how many is in the Purſe?

	1	$\frac{xx}{2} =$ the Pounds in the Purſe.
Then	2	$xx =$ the Pounds doubled.
And	3	$xx+x+20 = y$ by Subſtitution.
Then	4	$yy+y = 194040$
4 C □	5	$yy+y+{,}25 = 194040{,}25$
5 vu 2	6	$y+{,}5 = \sqrt{194040{,}25} = 440{,}5$
6 — ,5	7	$y = 440$
∴	8	$xx+x+20 = 440$ by Reſtitution.
8 — 20	9	$xx + x = 420$
9 C □	10	$xx + x +{,}25 = 420{,}25$
10 vu 2	11	$x+{,}5 = \sqrt{420{,}25} = 20{,}5$
11 — ,5	12	$x = 20$
by the 1ſt.	13	$\frac{xx}{2} = 200$ the Pounds in the Purſe.

No. 99.

Two Travellers ſet out at the ſame Time from two Cities which are 129 Miles aſunder, one of them goes 6 Miles every Day, and the other 2 Miles the firſt Day, $2\frac{1}{2}$ the ſecond, 3 the third, and ſo on, adding $\frac{1}{2}$ Mile to every Day's Journey. In what Time will they meet with one another, and how far will each have travelled ?

	1	$x =$ the Days each travelled.
Then	2	$6x =$ the Miles one travelled.
	3	$x - 1 =$ the Number of Terms.—1
$3 \times ,5$	4	$,5x - ,5$
	5	$,5x + 1,5 =$ the laſt Term.
	6	$\frac{,5xx + 3,5x}{2} + 6x = 129$
6×2	7	$,5xx + 3,5x + 12x = 258$
$\therefore$	8	$,5xx + 15,5x = 258.$
$8 \div ,5$	9	$xx + 31x = 516$
$9 \, C \, \square$	10	$xx + 31x + 240,25 = 756,25$
10 ѵѵ 2	11	$x + 15,5 = \sqrt{756,25} = 27,5$
$11 - 15,5$	12	$x =$ 12 the Number of Days each travelled.

One travels 72 Miles, and the other 57.

N. B. You may ſee the Reaſon of the 3d, 4th, 5th and 6th Steps by the Theorems in Page 68.

No. 100.

Two Merchants *A* and *B* go Partners, *B* brings 420 Pounds, and *A* receives out of the Gains 52 Pounds, and the Sum of both their Shares is 854 Pounds. How much did *A* bring, and how much did *B* receive out of the Gains?

	1	$x =$ *A*'s Stock.
Then	2	$x : 52 :: 420 : \frac{21840}{x}$

And	3	$x+420+52+\frac{21840}{x}=854$
$3\times x$	4	$xx+420x+52x+21840=854x$
$4-21840$	5	$xx+472x=854x-21840$
$5-854x$	6	$xx-382x=-21840$
6 C □	7	$xx-382x+36481=14641$
7 vv 2	8	$x-191=121$
$8+191$	9	$x=312$ *A*'s Stock.
And	10	$\frac{21840}{x}=70$ *B*'s Gain.

No. 101.

Two Farmers ſell two Sorts of Corn; *A* ſells 6 Buſhels, *B* receives in all 20 Crowns; now ſays *B* to *A*, if we add the Number of my Buſhels to the Number of your Crowns, the Sum will be 28; ſays *A* to *B*, and if I add the Square of my Crowns to the Square of your Buſhels, the Sum will be 424. How many Buſhels did *B* ſell, and how many Crowns did *A* receive?

	1	$x=$ *B*'s Buſhels.
	2	$y=$ *A*'s Crowns.
	3	$x+y=28$
	4	$xx+yy=424$
3 ⊙ 2	5	$xx+2xy+yy=784$
$5-4$	6	$2xy=360$
$4-6$	7	$xx-2xy+yy=64$
7 vv 2	8	$x-y=8$
$3+8$	9	$2x=36$
$9\div 2$	10	$x=18=$ *B*'s Buſhels.
$3-10$	11	$y=10=$ *A*'s Crowns.

No. 102.

A certain Man intends to travel as many Days as he has Crowns. It happens that every following Day of his Journey he had as many Crowns as he had the Day before, beſides two Crowns over and above; and when he came to his Journey's End he had in all 45 Crowns. How many had he at firſt?

	1	$x =$ the leaſt Term.
	2	$x =$ the Number of Terms.
Then	3	$3x - 2 =$ the greateſt Term.
∴	4	$4xx - 2x = 90$
$4 \div 4$	5	$xx - ,5x = 23,5$
5 C □	6	$xx - ,5x + ,0625 = 22,5625$
6 ɯ 2	7	$x - ,25 = 4,75$
7 + ,25	8	$x = 5 =$ the Number of Crowns he had at firſt, and Days travelled.

No. 103.

A and *B* owe betwixt them 174 Pounds, *A* pays eight Pounds a Day, and *B* pays the firſt Day one Pound, the ſecond Day two Pounds, the third three Pounds, and ſo on. In how many Days will they clear the Debt, and how much did each of them pay?

	1	$x =$ the Number of Days.
Then	2	$8x =$ what *A* pays.
And	3	$\frac{xx+x}{2} =$ what *B* pays.
2 + 3	4	$\frac{xx+x}{2} + 8x = 174$
4 × 2	5	$xx + 17x = 348$

5 C□	6	$xx + 17x + 72,25 = 420,25$
6 vv 2	7	$x + 8,5 = 20,5$
7 — 8,5	8	$x = 12$ the Number of Days.

No. 104.

Two Countrymen, *A* and *B*, ſell their Corn at different Prices; *A* ſells twenty Buſhels, and *B* received for one Buſhel as many Shillings as he ſold Buſhels. *A* perceives that if he had ſold as many Buſhels as *B* received Shillings, he ſhould have received 252 Shillings, but both together received 176. How many Buſhels did *B* ſell, and what Price had *A*?

	1	$x =$ *B*'s Buſhels.
	2	$xx =$ the Shillings *B* received.
∴	3	$xx : 252 :: 20 : \frac{5040}{xx}$
Then	4	$xx + \frac{5040}{xx} = 176$
4×xx	5	$xxxx + 5040 = 176xx$
5 ±	6	$xxxx - 176xx = -5040$
6 C□	7	$xxxx - 176xx + 7744 = 2704$
7 vv 2	8	$xx - 88 = 52$
Then	9	$xx = 36$
9 vv 2	10	$x = 6$ the Buſhels *B* ſold.
And	11	$\frac{5040}{xx} = 140$ the Shillings *A* received.

No. 105.

Find a Number, from whoſe Double if you ſubtract 12, the Square of the Remainder —1, ſhall be equal to nine times the Number?

	1	$x =$ the Number.
	2	$2x - 12$
$2 \odot 2$ & -1	3	$4xx - 48x + 143 = 9x$
$3 \pm$	4	$4xx - 57x = -143$
$4 \div 4$	5	$xx - 14{,}25x = -35{,}75$
5 C ▭	6	$xx - 14{,}25x + 50{,}765625 = 15{,}015625$
6 vv 2	7	$x - 7{,}125 = 3{,}875$
$7 + 7{,}125$	8	$x = 11 =$ the Number.

No. 106.

What two Numbers are thoſe whereof twice the firſt, with three times the ſecond will make ſixty; moreover twice the Square of the firſt, with three times the Square of the ſecond, will be 840?

	1	$x =$ the firſt.
	2	$y =$ the ſecond.
	3	$2x + 3y = 60$
	4	$2xx + 3yy = 840$
$3 - 2x$	5	$3y = 60 - 2x$
$5 \div 3$	6	$y = \frac{60 - 2x}{3}$
$6 \odot 2$	7	$yy = \frac{3600 - 240x + 4xx}{9}$
$4 - 2xx$	8	$3yy = 840 - 2xx$
$8 \div 3$	9	$yy = \frac{840 - 2xx}{3}$
7 & 9	10	$7560 - 18xx = 10800 - 720x + 12xx$
$10 \pm$	11	$30xx - 720x = -3240$
$11 \div 30$	12	$xx - 24x + 144 = 36$
12 C ▭	13	$xx - 24x + 144 = 36$
13 vv 2	14	$x - 12 = 6$
$14 + 12$	15	$x = 18$
by the 3d.	16	$y = 8$

No. 107.

A certain Number conſiſts of two Places, which is equal to four times the Sum of its Digits, and if to the Number you add eighteen, the Digits will be inverted. What is the Number?

	1	$y =$ the Unit. Place.
	2	$x =$ the Ten's Place.
Then	3	$10x + y = 4x + 4y$
And	4	$10y + x = 4x + 4y + 18$
$3 - 4x$	5	$6x + y = 4y$
$5 - y$	6	$6x = 3y$
$6 \div 3$	7	$2x = y$
Then	8	$10y + \frac{y}{2} = 6y + 18$
8×2	9	$21y = 12y + 36$
$9 - 12y$	10	$9y = 36$
$10 \div 9$	11	$y = 4$
And	12	$21x = 12x + 18$
$11 - 12x$	13	$9x = 18$
$13 \div 9$	14	$x = 2$

No. 108.

There is a certain Rectangle whoſe Length exceeds its Breadth by four, and its Area is ſixty. I demand the Length and Breadth?

	1	$x + 4 =$ the Length.
	2	$x =$ the Breadth.
1×2	3	$xx + 4x = 60$
$3 \; C \; \square$	4	$xx + 4x + 4 = 64$
$4 \; \omega \; 2$	5	$x + 2 = 8$

5—2

5 − 2	6	$x = 6$ the Breadth.
by the 1ſt.	7	$x + 4 = 10$ the Length.

No. 109.

There is a rectangled Parallelogram, whoſe Length and Breadth together equals 40, and the Area is 336. I demand the Sides?

	1	$x =$ the Length.
And	2	$40 - x =$ the Breadth.
1 × 2	3	$40x - xx = 336$
3 ±	4	$xx - 40x = -336$
4 C □	5	$xx - 40x + 400 = 64$
5 vu 2	6	$x - 20 = 8$
6 + 20	7	$x = 28$ the Length.
by the 2d.	8	$40 - x = 12$ the Breadth.

No. 110.

There is a Square whoſe Diameter exceeds its Side by 6, what is the Side of the Square?

	1	$x =$ the Side.
	2	$x + 6 =$ the Diameter.
Then	3	$2xx = xx + 12x + 36$
3 − xx	4	$xx = 12x + 36$
4 − 12x	5	$xx - 12x = 36$
5 C □	6	$xx - 12x + 36 = 72$
6 vu 2	7	$x - 6 = 8{,}485$
7 + 6	8	$x = 14{,}485$ the Side of the Square.
by the 2d.	9	$x + 6 = 20{,}485$ the Diameter.

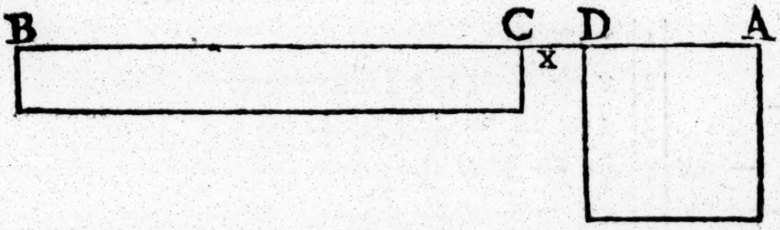
No. 111.

There is a Garden in form of a Rectangle, whoſe Length is three times its Breadth; this Garden is divided into two Areas, the Length of one is 18, and the Content of the Remainder is 120. I demand the Length and Breadth?

	1	$3x =$ the Length.
	2	$x =$ the Breadth.
	3	$18 =$ the Length of one Part.
	4	$3x - 18 =$ the Length of the other.
4×2	5	$3xx - 18x = 120$
$5 \div 3$	6	$xx - 6x = 40$
6 C □	7	$xx - 6x + 9 = 49$
7 vv 2	8	$x - 3 = 7$
$8 + 3$	9	$x = 10$ the Breadth.
by the 1ſt.	10	$3x = 30$ the Length.

No. 112.

Let the Line *A B* (of 70 Parts) be divided in *C*, ſo that *A C* = 22, and *B C* 48, it is required to divide the ſame Line again in another Point, as for Example, in *D*, ſo that the Rectangle *B*, *D*, *C*, may = the Square *A*, *D*.

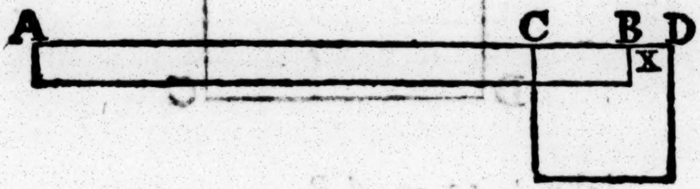


	1	$AC = 22$
	2	$BC = 48$
	3	$CD = x$
	4	$AD = 22 - x$
	5	$484 - 44x + xx = 48x$
5 ±	6	$xx - 92x = -484$
6 C □	7	$xx - 92x + 2119 = 1632$
7 vv 2	8	$x - 46 = 40{,}398$
	9	$x = 5{,}602$
by the 4th.	10	$22 - x = 16{,}398 = AD$.

No.

No. 113.

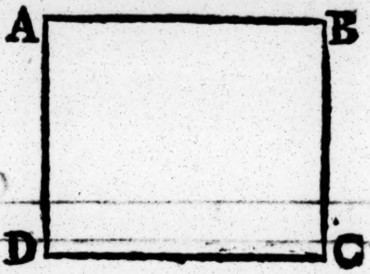
Let the Line AB be divided in C, ſo that $AC = 20$, and $CB = 4$, it is required to produce the Line AB to D, ſo that the Rectangle ABD may $=$ the Square CD?



	1	$AC = 20$
	2	$CB = 4$
	3	$BD = x$
	4	$CD = 4 + x$
	5	$16 + 8x + xx = 24x$
$5 \pm$	6	$xx - 16x = -16$
$6\ C \square$	7	$xx - 16x + 64 = 48$
$7\ \omega\ 2$	8	$x - 8 = 6{,}928$
	9	$x = 1{,}072$

No. 114.

In the Rectangle *A B C D*, the Difference of the greater Side *A B*, and of the leſſer Side *C D* is 12, but the Difference of the Squares of the Sides is 1680. What are the Sides?



	1	$x + 12 = AB$
	2	$x = BC$
1 ⊚ 2	3	$xx + 24x + 144$
2 ⊚ 2	4	xx
3 — 4	5	$24x + 144 = 1680$
5 — 144	6	$24x = 1536$
6 ÷ 24	7	$x = 64$
by the 1ſt.	8	$x + 12 = 76$

No. 115.

In the Rectangle *D E F G*, the Length is twice the Breadth, and the Sum of the Squares of the Length and Breadth is ten times the Sum of the Sides. *Quære* the Sides?

I

1 | $x = EF$

2 | $2x = DE$

Then 3 | $5xx =$ the Sum of the Squares.

And 4 | $30x =$ ten times the Sum of the Sides.

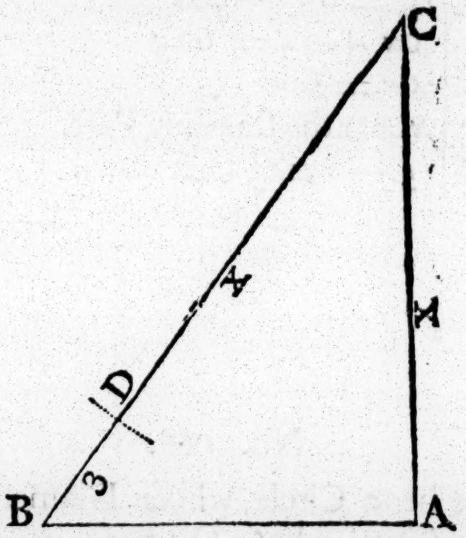
∴ 5 | $5xx = 30x$

$5 \div 5x$ 6 | $x = 6$

by the 2d. 7 | $2x = 12$

No. 116.

In the rectangle Triangle, ABC is given the Baſe, $AB = 9$, and the Difference of the other Sides, that is the Segment $BD = 3$. *Quære*, the Sides AC and BC?



1 | $x = AC$

2 | $x + 3 = BC$

3 | $6x + 9 = 81$ per Figure.

$3 - 9$ 4 | $6x = 72$

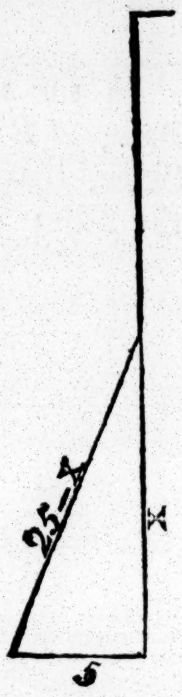
$4 \div 6$ 5 | $x = 12 = AC$

by the 2d. 6 | $x + 3 = 15 = BC$.

No. 117.

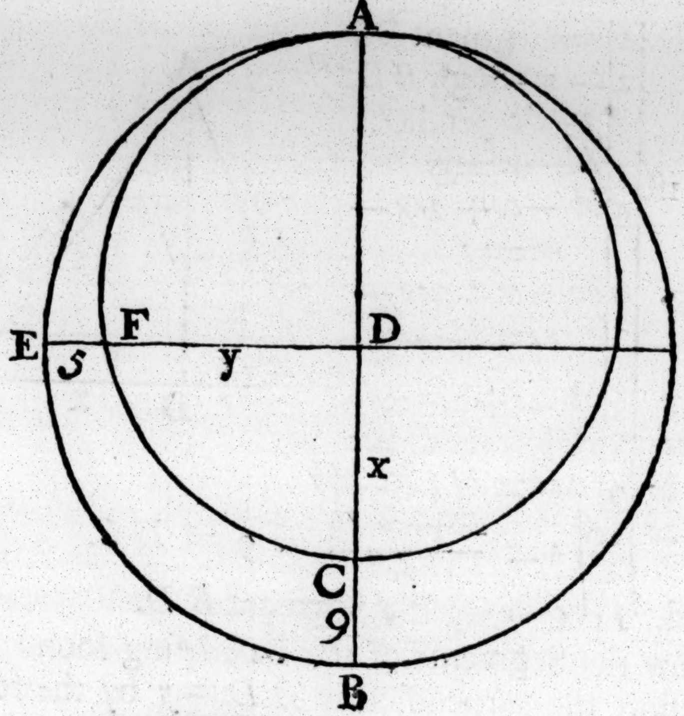
If a May-Pole be 25 Yards high, and by the Wind it is blown down, ſo that the Top-End ſtruck the Ground five Yards from the Foot of the Pole, I demand the ſtanding Part, and how much was blown down?

	1	$x =$ the ſtanding Part.
	2	$25 - x =$ the Part down.
	3	$xx + 25 = 625 - 50x + xx$
$3 - xx$	4	$25 = 625 - 50x$
$4 + 50x$	5	$50x + 25 = 625$
$5 - 25$	6	$50x = 600$
$6 \div 50$	7	$x = 12$ the ſtanding Part.
by the 2d.	8	$25 - x = 13$ Part down.



No. 118.

Let there be a Circle whoſe Diameter is *A B*, with another Circle whoſe Diameter is *A C*, touching the great Circle in the Point *A*, and from the Center of the greater Circle *D*, draw the Radius *D E*, at right Angles to *A B*, cutting the Periphery of the leſſer Circle in *F*; now there is given *B C* the Difference of the Diameters = 9, with the Segment *E F*, equal 5. *Quære* the Diameters?

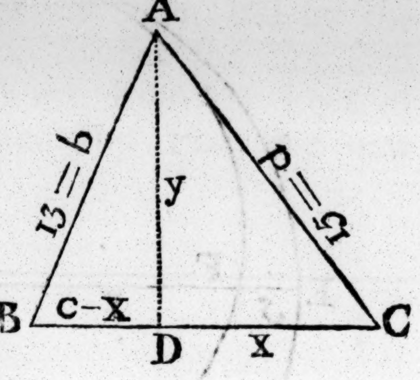


	1	$DC = x$
	2	$FD = y$
	3	$x : y :: y : \frac{yy}{x} = 9 + x$
	4	$5 + y = 9 + x$
$4 - 5$	5	$y = 4 + x$
$5 \circledcirc 2$	6	$yy = 16 + 8x + xx$
by the 3d.	7	$9x + xx = 16 + 8x + xx$
$7 - xx$	8	$9x = 16 + 8x$
$8 - 8x$	9	$x = 16$
$\therefore$	10	$x + 9 = 25 =$ the Radius *D B*.
And	11	$20{,}5 =$ the Radius of the leaſt Circle.

No. 119.

There is an acute angled Triangle, as *A B C*, whoſe Sides are known, *viz.* $AB = 13 = b$, and $BC = 14 = c$, and $AC = 15 = d$. I demand the Point of the Baſe, where a Perpendicular let fall from the Vertex or Top of the Triangle, ſhall cut it and the perpendicular Line?

	1	$x =$ Segment DC
	2	$c - x =$ Seg^t. BD.
	3	$y =$ Perpendic^r.
47 *E*. 1ſt	4	$dd - xx = yy$
	5	$bb - cc + 2cx - xx = yy$
4 & 5	6	$bb - cc + 2cx - xx = dd - xx$
$6 + xx$	7	$bb - cc + 2cx = dp$
$7 + cc$	8	$bb + 2cx = dd + cc$
$8 - bb$	9	$2cx = dd + cc - bb$
$9 \div 2c$	10	$x = \frac{dd + cc - bb}{2c} = 9 = DC$.
by 2d.	11	$c - x = 5$ the Segment BD.

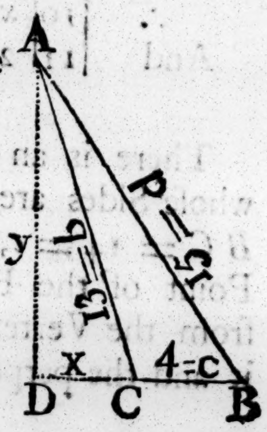


Now the Segments of the Baſe being found, you may find the perpendicular $AD = y$ by the fourth or fifth Steps.

No. 120.

There is an obtuſe-angled Triangle, *A B C*, whoſe three Sides are known, *viz*. $AB = 15 = d$, and $AC = 13 = b$, and $BC = 4 = c$. ; demand the Point without the Triangle, the Baſe being produced ſhall meet with a Perpendicular and the Perpendicular ?

	1	$x = DC$ Part of the Baſe produced.
	2	$y = AD$ the Perpendicular.
	3	$xx + 2cx + cc + yy = dd$
47 *Eu*. 1ſt.	4	$xx + yy = bb$
$3 - 4$	5	$2cx + cc = dd - bb$
$5 - cc$	6	$2cx = dd - bb - cc$



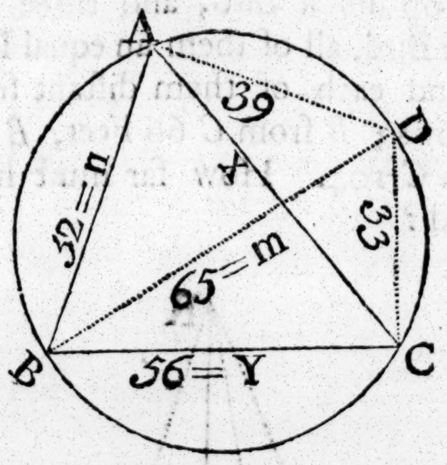
6 ÷ 2c | 7 | $x = \frac{dd - bb + cc}{2c} = 5 =$ the produced Part of the Baſe.

Now x being found, y may be found by the third or fourth Steps.

4 — xx | 8 | $yy = bb - xx$
8 vv 2 | 9 | $y = \sqrt{bb - xx} = 12$ the Perpendicular $A\,D$.

No. 121.

There is a Triangle whereof two Sides are known, as $A\,B = 52$, and $B\,C = 56$, about the ſame is circumſcribed a Circle, whoſe Diameter $= 65$. How much is the third Side of this Triangle, that is $A\,C$?



The Angles $B\,A\,D$, and $B\,C\,D$, are right Angles by 31ſt. *Euclid* 3d.

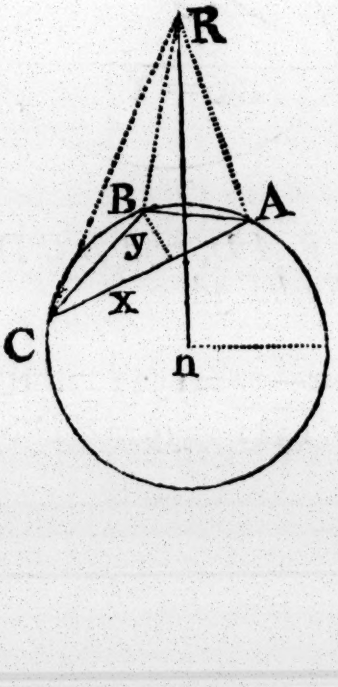
Then | 1 | $mm - nn = 1521 =$ the ☐ of $A\,D$.
And | 2 | $\sqrt{1521} = 39 = A\,D$ by 47e 1ſt.

	3	$mm - yy = 1089 = \square$ of *D C.*
	4	$\sqrt{1089} = 33 = DC$ by 47 *Euc.* 1ſt.
	5	$x =$ the Side required *A C.*
Then	6	$65x = 3900$
6 ÷ 65	7	$x = \frac{3900}{65} = 60$ the Length of the Side

For in a Quadrangle inſcribed in a Circle the interior oppoſite Angles together are equal to two right Angles, by 22d *Euclid* 3d. And if there be drawn two Diagonals, the Rectangle under the Diagonals is equal to the two Rectangles under the oppoſite Sides.

No. 122.

Suppoſe a May-Pole 156 Feet high, as *N R*, and upon the Top ſits a Bird, and three Men are to ſhoot at the Bird, all of them an equal Diſtance from the Pole, and each of them diſtant from one another as follows; *B* from *C* 66 Feet, *B* from *A* 50, and *A* from *C* 104. How far muſt he ſhoot that hits the Bird?



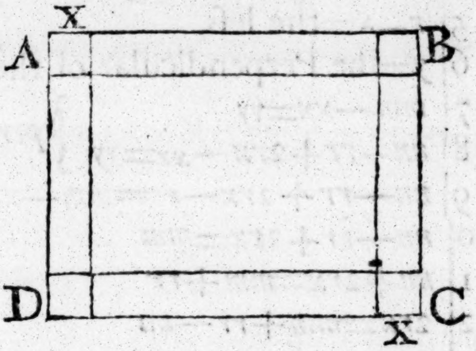
It is evident the Places of their ſtanding make a Triangle in a Circle?

	1	$m=66=CB$	
	2	$n=50=BA$	
	3	$r=104=CA$	
	4	$x=$the greater Segment.	
	5	$r-x=$the leſs.	
	6	$y=$the Perpendicular of the Triangle.	
Then	7	$mm-xx=yy$	*per* 47 1ſt. *Euc.*
And	8	$nn-rr+2rx-xx=yy$	
∴	9	$nn-rr+2rx-xx=mm-xx$	
$9+xx$	10	$nn-rr+2rx=mm$	
$10+rr$	11	$nn+2rx=mm+rr$	
$11-nn$	12	$2rx=mm+rr-nn$	
$12\div 2r$	13	$x=\frac{mm+rr-nn}{2r}=60\frac{12}{13}$	
by the 7th.	14	$\sqrt{mm-xx}=y=25\frac{5}{13}$	
∴	15	$x:y::156:65$ the ſemi-Diameter of the Circle.	

Then the Square of the Pole's Height added to the Square of the ſemi-Diameter, the Sum will be 28561, whoſe Square Root is 169; and ſo many Feet is each from the Bird.

No. 123.

A Perſon hath a right-angled Piece of Land, as *A B C D*, in Length forty Yards, and in Breadth thirty Yards. This he would convert into a Garden, but finding the ſame in the Winter to be annoyed with Water, he would have the ſame raiſed one Yard over the whole Superficies. Now it is reſolved to make a Ditch round about the ſame which ſhall be two Yards deep, ſo that the Ditch may remain of equal Breadth and equal Depth, till ſo much Earth come out of the ſame, as will raiſe the whole Superficies remaining within the Ditch one Yard higher. How broad muſt the Ditch be?



1 | $x =$ the Breadth ſought.
2 | $40 - 2x =$ the Length.
3 | $30 - 2x =$ the Breadth.
Then 4 | $12xx - 420x = -1200$
$4 \div 12$ 5 | $xx - 35x = -100$
5 C □ 6 | $xx - 35x + 306,25 = 206,25$
6 ⱱⱱ 2 7 | $x - 17,5 = \sqrt{206,25} = 14,361406$
∴ 8 | $x = 3,138594$ the Breadth ſought.

F I N I S.

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